Effect of electron inertia and electrical resistivity on Jeans Instability of Quantum plasma

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Abstract. Theanalytical description is presented for the influence of electron inertia and electrical resistivity on the Jeans instability magnetized quantum plasma. The QMHD fluid model is used toformulating the problem. The general dispersion relation is obtained using the normal mode analysis technique, which is reduced for both the parallel and perpendicular mode of propagation. The Jeans criterion of instability is modified by electron inertia, porosity, viscosity and quantum correction in the perpendicular of the magnetic field. From the graphical presentation we show that electron inertia hasstabilized the system with magnetized quantum plasma but in the absence of magnetic field and quantum correction, the electron inertia has a destabilizing influence on the growth rate of instability in the system. The condition of instability is affected by the presence of Alfven velocity, porosity and quantum parameter. The graphical presentation shows that the magnetic field and quantum correction has a stabilizing effect on the system.

INTRODUCTION

The quantum magnetohydrodynamic fluid model has attracted more attention in recent year due to their important applications in plasma physics, astrophysics and many other crucial phenomenon of the interstellar medium (ISM). The quantum effect start to play a vital role in high density and low temperature in the typical plasma environment [1]. The quantum plasma was first introduced by pine [2] and QMHD model was first developed by Hass [3]. The gravitational instability of the system is play an important role for the condensation and formation of astrophysical objects. The first prominent theory of gravitational condensation an astrophysics objects for the formation of stars was given by Jeans [4]. In this direction, the Jeans instability of self-gravitating magnetized rotating plasma is examined by Chandrasekhar [5]. In addition to this electron inertia parameter is play a significant role to understanding magnetic reconnection process and instability investigation of accelerated plasma. Many researchers [6-12] have investigate the influence of electron inertia with including various parameters, but no one studies the influence of electron inertia with magnetized quantum plasma. In this brief communication, we focus on finding the impact of electron inertia presence and absence of magnetic field and quantum correction on the growth rate of the system. The present result is helpful to understand the various astrophysical phenomenon.

EQUATION OF THE PROBLEM

The momentum transfer equation

$$\frac{\delta v}{\delta t} = -\frac{\nabla \delta p}{\rho} + \nabla \delta \phi + \vec{v}\left(\nabla^2 \vartheta - \frac{\vartheta}{K_1}\right) + \frac{1}{4\pi\rho}(\nabla \times h) \times H + \frac{\hbar^2}{4m_e m_i}\nabla \frac{(\nabla^2 \delta\rho)}{\rho} \quad (1)$$

$$\varepsilon\frac{\partial\delta\rho}{\partial t} = -\rho\nabla.v \quad (2)$$

$$\nabla^2\delta\phi + 4\pi G\delta\rho = 0 \quad (3)$$

$$\frac{\partial h}{\partial t} = \nabla\times(v\times H) + \eta\nabla^2 h + \frac{c^2}{\omega_{pe}^2}\frac{\partial}{\partial t}\nabla^2\vec{h} \quad (4)$$

$$\nabla.H = 0 \quad (5)$$

Where,$\nabla p = c_s^2\nabla\rho and c_s^2$ is modified sound speed due to quantum effects and $V^2 = H^2/4\pi\rho$, $v(v_x, v_y, v_z), \rho, p, \phi, H(0,0,H), \omega_{pe}, \vartheta, K_1, G, \varepsilon, \eta,\ \hbar$, denote respectively, the gas velocity, density, fluid pressure, gravitational potential, magnetic field, electron plasma frequency, viscosity, permeability, gravitational constant, porosity, resistivity and Plank's constant divided by $2\pi, m_e and m_i$ are the electron and ion mass respectively.

DISPERSION RELATION

Let us assume that all the perturbed quantities vary as

$exp\{i(k_x x + k_z z + \omega t)\}$(6)

Where $\sigma = i\omega$ is the growth rate of the perturbation, k_x and $k_z$are the wave numbers along x and z-direction to the magnetic field and $k_x^2 + k_z^2 = k^2$. After some algebraic manipulation the general dispersion relation is obtained solving equations (1)-(6) thus we obtained equation (7)

$$-a_1 D_1 D_2 D_3 + \left[\frac{k_x^2}{k^2}\left(\Omega_J^2 + \frac{\hbar^2 k^4}{4m_e m_i}\right)\left(\frac{k^2 V^2 a_1 D_2}{(\sigma f + \Omega_m)}\right)\right] = 0 \quad (7)$$

Where,

$$a_1 = (\sigma + \vartheta_k), \Omega_J^2 = (k^2 c^2 - 4\pi G\rho), \quad \Omega_m = (\eta k^2), \quad s = \left(\frac{\delta\rho}{\rho}\right), \quad D_1 = \left(a_1 + \frac{k^2 V^2}{(\sigma f + \Omega_m)}\right),$$

$$D_2 = \left(a_1 + \frac{k_z^2 V^2}{(\sigma f + \Omega_m)}\right),\ D_3 = \left(\varepsilon\sigma^2 + \varepsilon\sigma\vartheta_k + \Omega_J^2 + \frac{\hbar^2 k^4}{4m_e m_i}\right),\ \vartheta_k = \vartheta\left(k^2 + \frac{1}{K_1}\right), f = \left(1 + \frac{c^2 k^2}{\omega_{pe}^2}\right)$$

PARALLEL MODE OF PROPAGATION

In this case, we assume all the perturbations parallel to the magnetic field $(k_x = 0, k_z = k)$,the general dispersion relation (7) reduced and become

$$(\sigma + \vartheta_k)\left(\sigma + \vartheta_k + \frac{k^2 V^2}{\sigma f + \Omega_m}\right)^2\left(\varepsilon\sigma^2 + \varepsilon\sigma\vartheta_k + \Omega_J^2 + \frac{\hbar^2 k^4}{4m_e m_i}\right) = 0 \quad (8)$$

It is clear from the above dispersion relation that when the propagation is parallel to the magnetic field we get three factors. The first factor has a natural stability and the second factor in above equation is equated to zero gives,

$$\sigma^2 f + \sigma(f\vartheta_k + \Omega_m) + \Omega_m\vartheta_k + k^2 V^2 = 0 \quad (9)$$

In this equation (9) the growth rate of the system is modified due to resistivity, viscosity permeability and electron inertiabut the Jeans condition of instability is not affected byelectron inertia.

The third factor of equation (8) is equated to zero represent the modified gravitating mode in the presence of permeability, porosity, viscosity and quantum correction.

$$\sigma^2 + \sigma\vartheta_k + \frac{\Omega_J^2}{\varepsilon} + \frac{\hbar^2 k^4}{4m_e m_i\varepsilon} = 0 \quad (10)$$

The Jeans instability criterion is obtained from the constant term of equation (10) is given as,

$$\frac{4\pi G\rho}{\varepsilon k^2} > \frac{c_s^2}{\varepsilon} + \frac{\hbar^2 k^2}{4m_e m_i\varepsilon}$$

This condition is modified by porosity. The condition is identical to given by Ren et al [6] when thenumerical value of porosity is $\varepsilon = 1$.

PERPENDICULAR MODE OF PROPAGATION

We assume that all the perturbations perpendicular to the magnetic field($k_x = k, k_z = 0$), the general dispersion relation (7) reduces to the simple form to give,

$$\sigma^4 f + \sigma^3(2f\vartheta_k + \Omega_m) + \sigma^2\left\{\vartheta_k(f\vartheta_k + 2\Omega_m) + \frac{f}{\varepsilon}\left(\Omega_J^2 + \frac{\hbar^2 k^4}{4m_e m_i}\right) + k^2V^2\right\} + \sigma\left\{\left(\frac{\Omega_m}{\varepsilon} + \frac{\vartheta_k f}{\varepsilon}\right)\left(\Omega_J^2 + \frac{\hbar^2 k^4}{4m_e m_i}\right) + \vartheta_k(\Omega_m\vartheta_k + k^2V^2)\right\} + \frac{\vartheta_k\Omega_m}{\varepsilon}\left(\Omega_J^2 + \frac{\hbar^2 k^4}{4m_e m_i}\right) = 0 \qquad (11)$$

Now we take the non-dimensional form to equation (11), for showing the effect of different parameters on the growth rate of instability in the system,

$$\sigma^{*4}f^* + \sigma^{*3}\left\{2f^*\vartheta^*\left(k^{*2} + \frac{1}{K_1^*}\right) + \eta^*k^{*2}\right\} + \sigma^{*2}\left[\left(k^{*2} + \frac{1}{K_1^*}\right)\left\{f^*\left(k^{*2} + \frac{1}{K_1^*}\right) + 2\eta^*k^{*2}\right\} + \frac{f^*}{\varepsilon}(k^{*2} - 1 + Q^*k^{*2}) + k^{*2}V^{*2}\right] + \sigma^*\left[\left(\frac{\eta^*k^{*2}}{\varepsilon} + \frac{f^*\left(k^{*2} + \frac{1}{K_1^*}\right)}{\varepsilon}\right)(k^{*2} - 1 + Q^*k^{*2}) + \left(k^{*2} + \frac{1}{K_1^*}\right)\left\{\eta^*k^{*2}\left(k^{*2} + \frac{1}{K_1^*}\right) + k^{*2}V^{*2}\right\}\right] + \frac{\left(k^{*2} + \frac{1}{K_1^*}\right)}{\varepsilon}\eta^*k^{*2}(k^{*2} - 1 + Q^*k^{*2}) = 0 \qquad (12)$$

Where the various non-dimensional parameters are defined as,

$$\sigma^* = \frac{\sigma}{\sqrt{4\pi G\rho}}, k^* = \frac{kc_s}{\sqrt{4\pi G\rho}}, Q^* = \frac{\hbar^2 k_j^2}{4m_e m_i}, V^* = \frac{V\sqrt{4\pi G\rho}}{c_s}, \vartheta^* = \frac{\vartheta\sqrt{4\pi G\rho}}{c_s^2}, \eta^* = \frac{\eta\sqrt{4\pi G\rho}}{c_s^2},$$

$$\Omega_J^{*2} = (k^{*2} - 1), \quad K_1^* = \frac{K_1\sqrt{4\pi G\rho}}{c_s^2}, \qquad f^* = f\sqrt{4\pi G\rho} \quad (13)$$

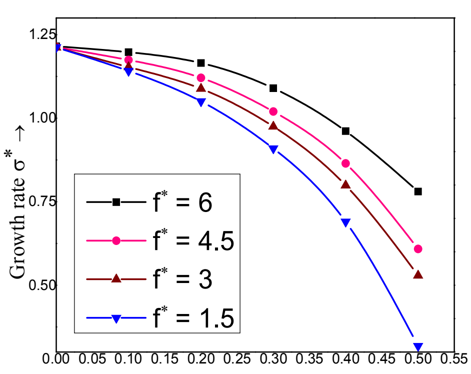

FIGURE 1. The normalized growth rate of the system versus wave number have shown with different values of electron inertia $f^* = 1.5, 3, 4.5, 6$ keeping the value of $Q^* = 0$ and other to be fixed.

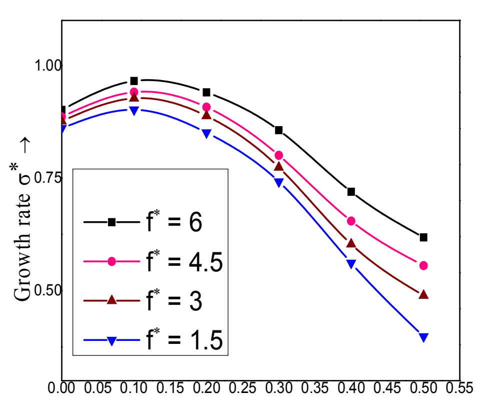

FIGURE 2. The normalized growth rate of the system versus wave number have shown with different values of electron inertia $f^* = 1.5, 3, 4.5, 6$ keeping the value of $Q^* = 1.5$ and other to be fixed.

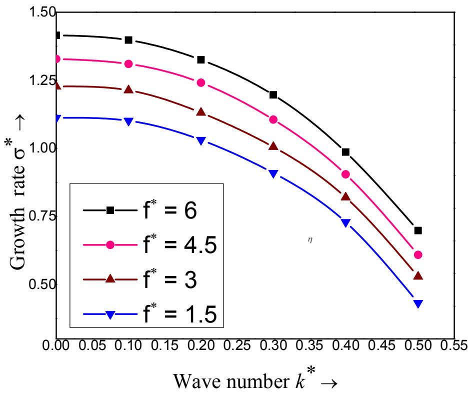

FIGURE 3. The normalized growth rate of the system versus wave number have shown with different values of electron inertia $f^* = 1.5, 3, 4.5, 6$ keeping the value of $V^* = 0$ and other to be fixed.

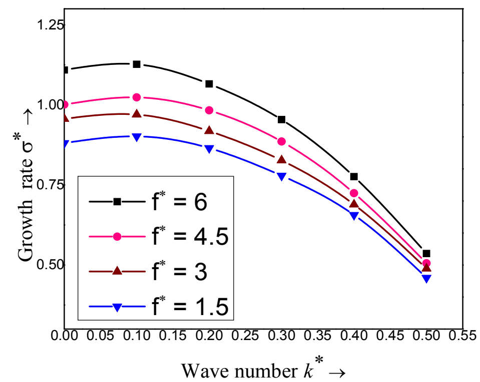

FIGURE 4. The normalized growth rate of the system versus wave number have shown with different values of electron inertia $f^* = 1.5, 3, 4.5, 6$ keeping the value of $V^* = 1.5$ and other to be fixed.

In figures 1-4, we have plotted the non-dimensional growth rate versus non-dimensional wave number for the different value of electron inertia and keeping the value of another parameter to be fixed. From the curve 1 we show that the electron inertia has a destabilizing effect but in curve 2 the electron inertia destabilizing effect is decrease presence in quantum correction. Similarly, in graphical presentation 3 and 4, we conclude that the electron inertia destabilizing influence is reduced presence in the magnetic field. The result is that the presence of a magnetic field and quantum correction is stabilized the system and reduced the destabilizing influence of electron inertia.

CONCLUSION

In the present study, the influence of electron inertia and electrical resistivity on the Jeans instability of viscous magnetized quantum plasmais investigated. The general dispersion relation is obtained using normal mode technique and which is reduced for the parallel and perpendicular mode of propagation. In the case of the parallel mode of propagation, we found that the growth rate of the system is modified by electron inertia and electrical resistivity but the condition of instability is not influenced by them. In the case perpendicular mode of propagation, we conclude that electron inertia is affected by the growth rate of the system but Jean's condition of instability is affected electrical resistivity and quantum correction. From the graphical presentation, we found that electron inertia has a destabilizing effect on the growth rate of the system but in the presence of magnetized quantum plasma is decrease the influence of electron inertia on the growth rate of the instability. The result will help to understand astrophysical problems.

REFERENCE

1. D. Kremp, Th. Bornath, M. Bonitz, and M. Schlanges, Physics Rev. E **60,** 4725 (1999).
2. D. Pines, J. Nucl. Energy C Plasma physics **2,** 5 (1961).
3. F. Hass, Physics of Plasmas **12,** 062117 (2005).
4. J. H. Jeans, Phil. Trans. Ray. Soc. London**199,** 1-53 (1902).
5. S. Chandrashekhar, "Hydrodynamics and hydromagnetic stability"Clarendon Press Oxford,**13,** (1961).
6. H. Ren, S. Wu, J. Cao, and P. K. Chu, Physics of Plasma **16,** 077101 (2009).
7. V. Kumar, D. L. Sutar and R.K. Pensia, Physics of Plasma **1953,** 060030 (2018).
8. C. Uberio, J. Plasma Fision Res. Ser. **8,** 823 (2009).
9. G. L. Karla and S. P. Talwar, In Annales d'Astrophysique **27,** 102 (1964).
10. P. Chatterjee and B. Das, Phy. Plasmas **11,** 3616 (2014).
11. D. L. Sutar and R.K. Pensia, Journal of Physics **836,** 012011 (2017).

12. S. Shaikh, A. Khan, and P.K. Bhatia, Physics Latter A **372,** 1451 (2008).