Jeans Instability of Quantum Plasma under the Influence of Hall Effect

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Abstract. We investigate the Jeans instability of magnetized quantum plasma including the effect of Hall current. The dispersion relation and numerical calculation have been performed QMHD model and normal mode analysis technique. The general dispersion relation is further reduced for the longitudinal and transverse to the direction of the magnetic field. In the case of longitudinal propagation, the Jeans criterion of instability is modified by quantum correction and porosity but the growth rate of instability is influenced by Hall current. From the curve, we find that the magnetic field and quantum correction have stabilizing effect while the electrical resistivity has a destabilizing influence on the system.

INTRODUCTION

The gravitational collapse of astrophysical objects is the fundamental key to the formation of comets, planets, stars, and galaxies. The first significant criterion of instability is given by James Jeans [1]. According to Jeans criterion, an infinite homogeneous self-gravitating atmosphere is unstable for all wave number k is less than the Jeans' wave number $k_j = \sqrt{G\rho}/_S$, where ρ is the density, *s* is the velocity of sound in gas and *G* is the gravitational constant. The present paper based on the quantum magneto-hydrodynamics (QMHD) fluid model. The quantum plasma is first examined by Pines [2] and the QMHD fluid model is given by Hass [3]. In a few years, the study of self-gravitational instability of inhomogeneous gaseous plasma with Hall current has attracted the attention of many researchers, due to its importance in astrophysics and cosmology. Many authors have discussed the problem of gravitational instability of inhomogeneous gaseous plasma with Hall current along with other parameters [4 - 12]. In the present work, we have investigated the problem of Jeans instability of quantum plasma with Hall current. The present research can serve as theoretical support to understanding various cosmological problems.

EQUATION OF THE PROBLEM

The momentum transfer equation

$$\frac{\delta v}{\delta t} = -\frac{\nabla \delta p}{\rho} + \nabla \delta \varphi + \left(\vartheta \nabla^2 - \frac{\vartheta}{K_1}\right) v + \frac{1}{4\pi\rho}(\nabla \times h) \times H + \frac{\hbar^2}{4m_e m_i} \nabla \frac{(\nabla^2 \delta\rho)}{\rho} \quad (1)$$

The equation of continuity

$$\varepsilon \frac{\partial \delta\rho}{\partial t} + \rho \nabla . v = 0 \quad (2)$$

The Poisson's equation for a self-gravitational potential

$$\nabla^2 \delta\varphi = -4\pi G\delta\rho \tag{3}$$

The induction equation for a magnetic field

$$\frac{\partial h}{\partial t} = \nabla \times (v \times H) + \eta\nabla^2 h - \frac{C}{4\pi Ne}[\nabla \times \{(\nabla \times h) \times H\}] \tag{4}$$

The Gauss's law for magnetism

$$\nabla . H = 0 \tag{5}$$

Where, $v(v_x, v_y, v_z), \rho, p, \varphi, H(0,0,H), \eta, K_1, N, c, e, G, \varepsilon, \hbar,$ denote the fluid velocity, density, fluid pressure, gravitational potential, magnetic field, resistivity, permeability, electron number density, the speed of light, electronic charge, gravitational constant, porosity and Plank's constant divided by $2\pi, m_e and m_i$ are the electron and ion mass respectively, $p = \rho c_s^2 and c_s^2$ is the modified sound speed due to the quantum effects.

DISPERSION RELATION

Let us assume that all the perturbed quantities vary as

$$exp\{i(k_x x + k_z z + \omega t)\} \tag{6}$$

Where $\sigma = i\omega$ is the growth rate of the perturbation, $V = \frac{H}{\sqrt{4\pi\rho}}$ is the Alfven velocity, and k_x and k_z are the wave numbers along x and z-direction to the magnetic field and $k_x^2 + k_z^2 = k^2$. After some numerical calculation, the general dispersion relation is obtained solving equations (1-6) and we get the dispersion relation (7)

$$\left(\sigma + \vartheta_k + \frac{k^2 V^2 A_1}{A_2}\right)\left[\left(\sigma + \vartheta_k + \frac{k_z^2 V^2 A_1}{A_2}\right)(\sigma^2 + \sigma\vartheta_k + \Omega_I^2)\right] + \left(\frac{k^2 k_z^2 V^2 A_3}{A_2}\right)^2 (\sigma^2 + \sigma\vartheta_k + \Omega_I^2)$$
$$- k_x^2 k_z^2 k_z V^2 \frac{A_3^2}{A_2^2}\Omega_I^2 + \left(\frac{k_x^2 V^2 A_1}{A_2}\right)\left(\sigma + \vartheta_k + \frac{k_z^2 V^2 A_1}{A_2}\right) = 0 \tag{7}$$

We have made following submissions,

$$\Omega_I^2 = \left(\Omega_J^2 + \frac{\hbar^2 k^4}{4 m_e m_i}\right), \Omega_J^2 = (k^2 c_s^2 - 4\pi G\rho), \quad \Omega_m = (\eta k^2), \quad s = \left(\frac{\delta\rho}{\rho}\right), \quad A_1 = (\sigma + \Omega_m),$$
$$A_2 = A_1^2 + A_3^2 k^2 k_z^2, \qquad A_3 = \frac{cH}{4\pi Ne}, \qquad \vartheta_k = \vartheta\left(k^2 + \frac{1}{K_1}\right)$$

LONGITUDINAL MODE OF PROPAGATION

In this case, we assume all the perturbations longitudinal to the magnetic field $(k_x = 0, k_z = k)$. Hence, the general dispersion relation (7) reduces to

$$\left(\sigma^2 + \sigma\vartheta_k + \frac{\Omega_I^2}{\varepsilon}\right)\left[\left(\sigma + \vartheta_k + \frac{k^2 V^2 A_1}{A_2}\right)^2 \left(\frac{k^4 V^2 A_3}{A_2}\right)^2\right] = 0 \tag{8}$$

It is clear from the above dispersion relation that when the propagation is in the longitudinal to the magnetic field it has two factors. The first factor has a natural stability and the second factor of equation (8) is equated to zero represent the modified gravitating mode due to the presence of viscosity, resistivity, magnetic field and Hall current.

$$\sigma^3 + \sigma^2(\vartheta_k + 2\Omega_m) + \sigma(\Omega_m^2 + k^4 A_3^2 + 2\vartheta_k\Omega_m + k^2 V^2) + \vartheta_k\Omega_m^2 + \vartheta_k k^4 A_3^2 + k^2 V^2 \Omega_m + k^4 V^2 A_3 = 0 \tag{9}$$

The above equation (9) shows that the growth rate of the instability of the system is modified due to resistivity, viscosity, magnetic field and Hall current but the Jeans criteria of instability are not affected by Hall current.

TRANSVERSE MODE OF PROPAGATION

In this case, the perturbations are taken to be transverse to the magnetic field $(k_x = k, k_z = 0)$. The general dispersion relation (7) reduces to

$$\left(\sigma+\vartheta_k+\frac{k^2V^2}{A_1}\right)\left[(\sigma+\vartheta_k)\left(\sigma^2+\sigma\vartheta_k+\frac{\Omega_I^2}{\varepsilon}\right)+\frac{k^2V^2}{A_1}\right]=0 \tag{10}$$

The above equation is two component the first component is show the stability is analyzed in previous papers. Now in the second component of equation (10) gives

$$\sigma^4+\sigma^3(2\vartheta_k+\Omega_m)+\sigma^2(2\vartheta_k\Omega_m+\vartheta_k^2)+\sigma\left\{\left(\vartheta_k^2\Omega_m+\vartheta_k\left(\frac{\Omega_J^2}{\varepsilon}+\frac{\hbar^2k^4}{4m_em_i\varepsilon}\right)+\frac{\Omega_J^2}{\varepsilon}+\frac{\hbar^2k^4}{4m_em_i\varepsilon}\right)\right\}$$
$$+\Omega_m\left(\frac{\Omega_J^2}{\varepsilon}+\frac{\hbar^2k^4}{4m_em_i\varepsilon}\right)(1+\vartheta_k)+k^2V^2$$
$$=0 \tag{11}$$

In the above equation is modified by all parameters. Now we take the non-dimensional form to equation (11), for showing the effect of different parameters on the growth rate of Jeans instability in the system.

$$\sigma^{*4}+\sigma^{*3}(2\vartheta_k^*+\eta^*k^{*2})+\sigma^{2*}(2\vartheta_k^*\eta^*k^{*2}+\vartheta_k^{2*})+\sigma^*\left\{\vartheta_k^{2*}\eta^*k^{*2}+\vartheta_k^*\left(\frac{k^{*2}-1}{\varepsilon}+\frac{Q^*k^{*2}}{\varepsilon}\right)+\frac{k^{*2}-1}{\varepsilon}+\frac{Q^*k^{*2}}{\varepsilon}\right\}$$
$$+(1+\vartheta_k^*)\eta^*k^{*2}\left(\frac{k^{*2}-1}{\varepsilon}+\frac{Q^*k^{*2}}{\varepsilon}\right)+k^{*2}V^{*2}$$
$$=0 \tag{12}$$

Where the various non-dimensional parameters are defined as,

$$\sigma^*=\frac{\sigma}{\sqrt{4\pi G\rho}},k^*=\frac{kc_s}{\sqrt{4\pi G\rho}},\quad Q^*=\frac{\hbar^2k_j^2}{4m_em_i},\quad V^*=\frac{V\sqrt{4\pi G\rho}}{c_s},\quad \vartheta^*=\frac{\vartheta\sqrt{4\pi G\rho}}{c_s^2},\quad \eta^*=\frac{\eta\sqrt{4\pi G\rho}}{c_s^2},$$
$$\Omega_J^{*2}=(k^{*2}-1)\vartheta_k^*=\vartheta^*\left(k^{*2}+\frac{1}{K_1^*}\right),\quad K_1^*=\frac{K_1\sqrt{4\pi G\rho}}{c_s^2},$$

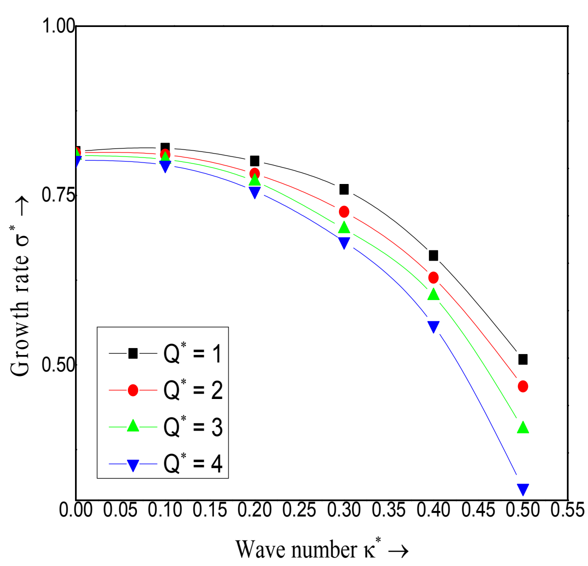

FIGURE 1. The normalized growth rate σ^* in the transverse mode, is plotted against wavenumber k^* for different values of the quantum correction$Q^*=1,2,3,4$,keeping the value of other parameters arefixed.

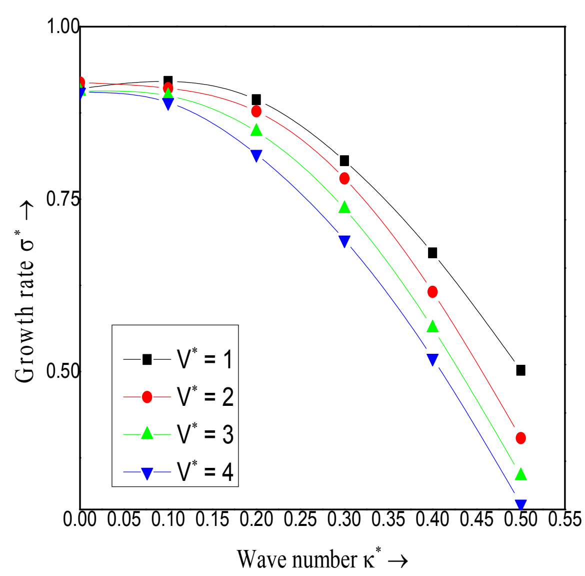

FIGURE2. The normalized growth rate σ^* in the transverse mode, is plotted against wavenumber k^* for different values magnetic field $V^*=1,2,3,4$, keeping the value of other parameters are fixed.

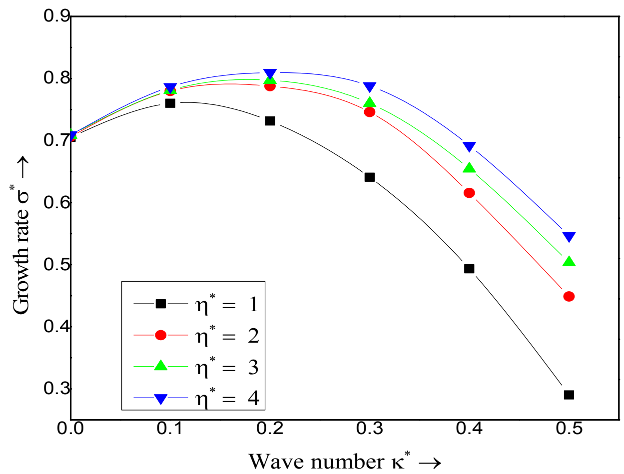

FIGURE 3. The normalized growth rate σ^* in the transverse mode, is plotted against wavenumber k^* for different values of electrical resistivity $\eta^* = 1, 2, 3, 4$, keeping the value of other parameters are fixed.

From the curves, we conclude that when the value of quantum correction and the magnetic field is increasing the pick value of growth rate of instability is decreasing while the value of resistivity increases than the growth rate of instability also increases. It is found that form the graphical presentation the quantum correction and magnetic field both the stabilizing influence and the electrical resistivity gives destabilizing impact on the growth rate of the system in a transverse mode of propagation.

CONCLUSION

In the recent work, we have analyzed the self-gravitational instability inhomogeneous gaseous plasma with Hall current, electrical resistivity, viscosity, magnetic field and quantum correction using normal mode analysis technique. We observed that the Hall current modified the growth rate of instability only longitudinal mode of propagation but the transverse mode is unaffected by Hall current. The Jeans condition of instability is modified by quantum correction and porosity. From the curve, it is clear that the magnetic field and quantum correction has a stabilizing effect and the resistivity has a destabilizing effect on the growth rate of the system in the transverse mode of propagation. It is hoped that the obtained results will support to understand the various astrophysical problems.

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