Linear Undulator Brightness With Error In Undulator Period

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Abstract. In this paper we study the spectral brightness of the radiation emitted by electrons moving in a linear undulator field with an error in the undulator period. The analysis is based on the evaluation of the Lienard-Wiechart potential retaining the condition of far field limit.

INTRODUCTION

In recent years there are several studies relating to emission of radiation in undulator magnets. A beam of relativistic electron undergoes transverse oscillations in the undulator magnets. The amount and spectrum of these oscillations is known as the spontaneous emission and is an important parameter in the startup and optimization of a free electron laser device. There are interests in evaluating the spontaneous emission in undulator magnets as it provides several new features such as betatron oscillation, angular separation, waveguide effects, beam energy and beam emittance spread.

Undulator and wigglers are the central components of free electron laser and the inevitable errors of actual magnetic wiggler causes a degraded performance of the free electron laser [1-6] with respect to performance obtained from ideally modeled magnetic wigglers. The design and analysis of a free electron laser typically assumes that the wiggler field to be described by its sinusoidal form. However the process of fabrication and assembly of wiggler magnets results in several sources of error that compels the field to vary from its ideal sinusoidal form by some small error. This error induces beam wander that allows transverse displacement of the electron beam from the magnetic axis and increase with the axial length of the wiggler. For long wigglers the effect of field errors are extremely important as it destroys the FEL interaction due to reduction in overlap of the electron pulse with the radiation field and leads to a loss of radiation gain. There are both theoretical and numerical studies of undulator field errors to predict the importance of these errors. The effects of the wiggler field errors are two fold .First the longitudinal velocity fluctuation which moves the electron beam away from resonance and second the transverse trajectory wander that causes the centroid of the beam to move away from the radiation beam.

In this paper we have reconsidered the theory of undulator brightness with inclusion of an error in the undulator period. Only planer undulators are discussed. In section two we have discussed the theory to derive an analytical expression for the electron beam trajectory that leads to an analytical expression for the radiation spectrum. This paper we have reconsidered the theory of undulator brightness with inclusion of an error in the undulator period. Only planer undulators are discussed. In section two we have discussed the theory to derive an analytical expression for the electron beam trajectory that leads to an analytical expression for the radiation spectrum .We follow numerical method to obtain the solution of the beam trajectory that allows a check and validity of the analytical theory.

THEORETICAL CALCULATION FOR UNDULATOR BRIGHTNESS

The Brightness i.e. energy radiated per unit solid angle and unit frequency interval by an electron moving in a magnetic field is calculated from the lienard-wiechart integral [7]

$$\frac{d^2 I}{d\omega d\Omega} = \frac{e^2\omega^2}{4\pi^2 c}\left|\int_0^\infty dt\left[n\times\left(n\times\beta\right)\right]\exp\left[i\omega\left(t-\frac{\hat{n}.\vec{r}}{c}\right)\right]\right|^2 \tag{1}$$

Where e is electron charge is the velocity of light in vacuum, ω is the radiation frequency, $\vec{r}$ is the electron trajectory, $\beta =$ electron reduced velocity, $\hat{n}$ is the observation unit vector. The integration over time in above equation is carried out from 0 to T, where $T = L/c\beta_z$ and L is the undulator length. We assume that on -axis field of a planar undulator with an error in the undulators period is given by[8]

$$B = 0, \hat{y}B_0 \sin\left(k_n z\right), 0$$

Where $\lambda_n = \lambda_u + \delta_n$ and λ_u defines the undulator wavelength and δ_n is error in undulator period. The trajectory of the electron is determined through the Lorentz equation. This gives

$$\frac{d^2 z}{dt^2} = \frac{eB_0}{m_0 c\gamma}\beta_x \sin\left[\frac{2\pi z}{\lambda_n}\right]$$

$$\frac{d^2 x}{dt^2} = -\frac{eB_0}{m_0\gamma}\beta_z \sin\left[\frac{2\pi z}{\lambda_n}\right] \tag{2}$$

To solve Eq(2) analytically, we assume at t=0, β_x=0.This helps to evaluate Eq.2.a and Eq 2.b yields an analytical expression,

$$\beta_x = \frac{K}{\gamma}\eta_n \cos\left[\Omega_n t\right] \quad , \quad \Omega_n = \frac{2\pi c}{\lambda_n} \tag{3}$$

The electron longitudinal velocity is obtained from the energy conservation rule. This gives

$$\beta_z = \beta^* - \frac{K^2\eta^2}{4\gamma^2}\cos\left[\Omega_n t\right] \tag{4}$$

Where

$$\beta^* = 1 - \frac{1}{2\gamma^2}\left[1 + \frac{K^2\eta_n^2}{2}\right]$$

The particle trajectory x, z are obtained by integrating expression of β_x and β_z respectively.
After mathematical computation expression for particle trajectory comes as follow

$$x = \frac{cK\eta_n}{\gamma\Omega_n}\sin\left[\Omega_n t\right] \tag{5}$$

$$z = \beta^* ct - \frac{K^2\eta_n^2 c}{8\gamma^2\Omega_n}\sin\left(2\Omega_n t\right) \tag{6}$$

In Eq(3-6), Ω_n is the undulator frequency with error in undulator wavelength and $\eta_n = \frac{\lambda_n}{\lambda_u}$ .K defines the undulator parameter. Following the electron trajectory, Eq(1) for the undulator brightness can be simplified for on axis emission. The exponential term, after a few algebraic steps, reads

$$\exp\left[\iota\omega\left(t-\frac{\hat{n}\cdot\vec{r}}{c}\right)\right]=\sum_{m}\exp\left(i\Omega_{n}t\left[\frac{\omega}{\omega_{1}}\sigma_{n}-m\right]\right)J_{m}(a,b)$$

(7)

Where $J_m(a,b)$ is a class of generalized Bessel function with arguments [9, 10]

$$a=\frac{\omega}{\Omega_{u}}\frac{K}{\gamma^{2}}\gamma\theta 2K\cos(\phi)(1+2\varepsilon_{n})$$

(8)

Where

$$\varepsilon_{n}=\delta_{n}/\lambda_{u}$$

$$\omega_{1}=\frac{2\gamma^{2}\Omega u}{1+\gamma^{2}\theta^{2}+K^{2}/2}$$

$$\sigma_{n}=1+\frac{\varepsilon_{n}\left[\gamma^{2}\theta^{2}+\left(1+\frac{3}{2}K^{2}\right)\right]}{\gamma^{2}\theta^{2}+\left(1+K^{2}/2\right)}$$

Collecting the terms and substituting the results for the triple vector product, the brightness can be expressed as

(9)

Where

$$I=\int_{0}^{T}\exp\left\{i\Omega_{n}t\left(\frac{\omega}{\omega_{1}}\sigma_{n}-n\right)\right\}dt$$

$$T_{n}^{x}=\theta\cos(\phi)J_{n}(a,b)-\eta_{n}\frac{K}{2\gamma}\left[J_{n+1}(a,b)+J_{n-1}(a,b)\right]$$

$$T_{n}^{y}=\theta\sin(\phi)J_{n}(a,b)$$

$$T_{n}^{z}=\frac{\eta_{n}K}{2\gamma}\theta\cos(\phi)\left[J_{n+1}(a,b)+J_{n-1}(a,b)\right]-\theta^{2}J_{n}(a,b)$$

RESULTS AND DISCUSSION

We have ploted the transverse particle trajectories for K=1, λ_u =5cm and $\gamma = 20$.Figure (1.a) plots numerically and analytical solution without error .The effects of error of 1% on particle trajectory. The analytical and numerical solution match throughout the undulator length. Intensity is also plotted for analytical and numerical solution which is shown in Figure (1.b).This justifies the assumption. The effects of the error in undulator period are visible towards the end of the interaction region. From calculation it is reflected that intensity will drops for higher value of error in undulator length.

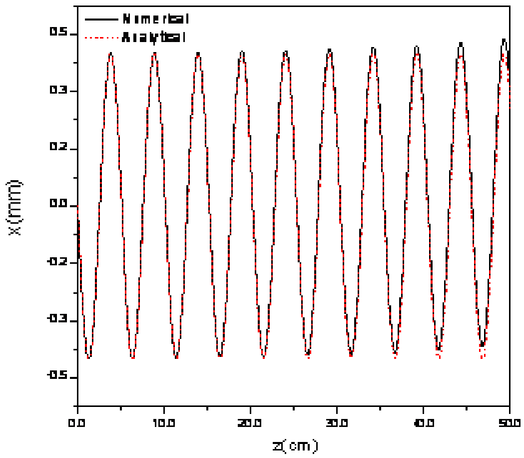

FIGURE 1.a Numerical and Analytical trajectory without error

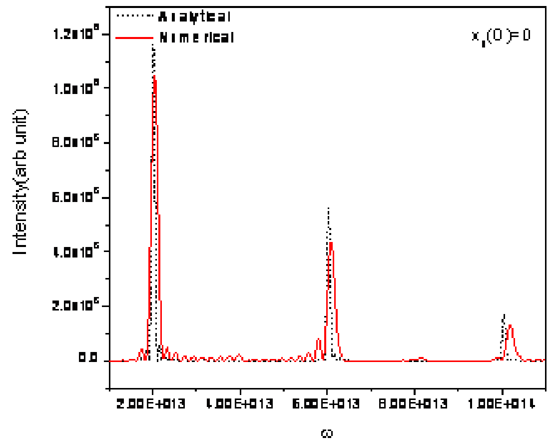

FIGURE 1.b Numerical and Analytical Intensity without error

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