

Effect of Kappa Distribution Function on Kinetic Alfvén Waves Instability in Dusty Magneto-Plasma

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Abstract. We have, present work on kinetic Alfvén waves instability in dusty plasma. The expression for the dispersion relation, growth rate and growth length of the kinetic Alfvén waves are derived using the particle aspect analysis in auroral acceleration region. Our purpose in this paper is to be investigating the effect of kappa distribution function with dusty plasma on kinetic Alfvén waves. The results of the work are consistent for Alfvén wave in dusty plasma are applicable of the magnetospheric and astrophysical in auroral acceleration region.

Keywords: kinetic Alfvén waves, Kappa distribution function, auroral acceleration, magnetosphere,

INTRODUCTION

The Kinetic Alfvén wave is the "Low Frequency Electromagnetic wave" which propagates obliquely to the magnetic fields while the magnetohydrodynamics Alfvén wave is obtained called a "High Frequency" wave by J.V. Hollweg (1999)¹. Kinetic Alfvén waves are of great importance in laboratory and space plasmas. These waves play an important role in energy transport, in driving field-aligned currents, in particle acceleration and heating and in explaining inverted V structures in magnetosphere-ionosphere coupling and in solar flares and the solar wind by A. Hasegawa et al., (1975)² & C.K. Goertz et al., (1984)³. The magnetohydrodynamic (MHD) Alfvén wave is converted to kinetic Alfvén waves via the ion finite Larmor radius effect by S.P. Duan et al., (2005)⁴. Kinetic Alfvén waves are accompanied by a compressive magnetic field and a parallel electric field. Kinetic Alfvén waves have been implicated in a wide variety of geophysical processes from the ionosphere to the solar corona by R.J. Leaman et al., (1999)⁵ & J.R. Wygant et al., (2002)⁶. It has also been observed simultaneously by polar and FAST satellite in the plasma sheet boundary layer at 4-6 RE (RE is the earth radius) measurements from the polar spacecraft show the existence of small scale Alfvén waves that carry a large net Poynting flux along magnetic field lines towards the earth. The small scale spikes have electric field amplitudes up to 300 mV m⁻¹ and associated magnetic field variations between 0.5 and 5nT. The analysis has so that the larger scale Alfvén waves have periods of ~20-60s and carry enough Poynting flux to explain the generation of the most intense auroral structures observed in the polar ultraviolet imager data set by J.R. Wygant et al., (2002)⁶. Low frequency plasma waves can control the dynamics of the ions in collision free plasmas and play a very important role in the formation and behavior of this region. The cusp region of the terrestrial magnetosphere play an important role in the transfer of energy from the solar wind to the ionosphere, since the cusp magnetic field lines directly can connected the solar wind with the earth polar regions by J. Blecki et al., (2005)⁷. The fast satellite crossing the polar cusp at 2000 km altitude has observed the electrostatic emissions at a broad range of frequencies by M. Bouhram et al., (2002)⁸.

Dusty plasmas are ionized gases in the presence of micro-particles and often are termed as Complex plasmas or colloidal plasmas by V.Jatenco-Pereira et al., (2014)⁹. Dusty plasma is generally a combination of normal electron ion plasma with an additional charged component of micron or submicron-sized particulates. It is likely that, 99% of the matter in our universe is in the form of plasma in which dust is one of the omnipresent ingredients by M.S.A. Khan et al., (2013)¹⁰.

DISTRIBUTION FUNCTION

To determine the dispersion relation and associated current, the distribution function of bi-kappa form is used;

$$F = \frac{1}{\pi^{3/2}} \frac{\Gamma(k+1)}{k^{3/2} \Gamma(k-\frac{1}{2}) V_{T\perp}^2 V_{T\parallel}^2} \left[1 + \frac{V_{\perp}^2}{k V_{T\perp}^2} + \frac{V_{\parallel}^2}{k V_{T\parallel}^2} \right]^{-(k+1)} \dots\dots(1)$$

In the equation (1) $V_{T\perp}$ and $V_{T\parallel}$ are thermal velocity related to the mass m and the temperatures T_{\perp} and T_{\parallel} respectively perpendicular and parallel to the magnetic field by

$$V_{T\perp}^2 = \left[\frac{k-3/2}{k} \frac{2K_{\beta} T_{\perp}}{m} \right]$$

And
$$V_{T\parallel}^2 = \left[\frac{k-3/2}{k} \frac{2K_{\beta} T_{\parallel}}{m} \right]$$

The k -Lorentz distribution has been introduced as more suitable for modeling magnetized plasma.

Temperature anisotropy $A = \frac{T_{\perp\alpha}}{T_{\parallel\alpha}}$ and $R_{\alpha} = \frac{q_{\alpha} B_0}{m_{\alpha} c}$ and $\omega_{p\alpha} = \left(\frac{4\pi n_{\alpha} e^2}{m_{\alpha}} \right)^{1/2}$ are, respectively, the gyrofrequency for electrons and ions. The kappa dispersion function

$$z_k(\xi) = \frac{1}{\pi^{1/2} k^{1/2}} \frac{\Gamma(k)}{\Gamma(k-1/2)} \int_{-\infty}^{\infty} \frac{dx (1+x^2/k)^{-k}}{(x-\xi_{\alpha})}$$

Where
$$\xi = \left(\frac{\omega - R_{\alpha}}{k V_{T\parallel\alpha}} \right)$$

DISPERSION RELATION

The perturbed density or non-resonant particles $n_{i,c,d}$ is given as

$$n_{i,c,d} = \int_0^{\infty} 2\pi V_{\perp} dV_{\perp} \int_{-\infty}^{\infty} dV_{\parallel} n_i(r, t)$$

The expression of n_i is used for non-resonant particles, which has been adopted for evaluated by Baronia & tiwari (14) as:

$$n_i(r, t) = F(V) \sum_{-\infty}^{\infty} J_n(\mu) \sum_{-\infty}^{\infty} J_l(\mu) \frac{q}{m} \left[\phi_1 - \frac{V_{\parallel} K_{\parallel}}{\omega} (\phi_1 - \psi_1) \times \frac{K_{\perp}^2}{a_n^2} + \frac{K_{\parallel}^2}{A_n^2} \psi_1 \right] \cos \xi_{nl} \dots\dots\dots(2)$$

Where
$$\mu = \frac{K_{\perp} V_{\perp}}{\Omega} \left[1 + \frac{3}{4} \frac{\tilde{E}(y)}{\Omega^2} \right],$$

$$A_n = K_{\parallel} V_{\parallel} - \omega + n\Omega + K_{\perp} \tilde{\Delta},$$

$$\tilde{\Delta} = \frac{-\tilde{E}(y)}{\Omega} \left[1 + \frac{E(y)}{E(y)} \frac{1}{4} \left(\frac{V_{\perp}}{\Omega} \right)^2 + \dots\dots \right],$$

$$a_n^2 = A_n^2 - \Omega^2,$$

$$\xi_{nl} = K_{\perp} Y + K_{\parallel} Z - \omega t + (n-l)(\Omega t - \theta)$$

θ is the initial phase of the charge particles, n and l are running symbols and integer. With the help of eq.(1) and (2) the average densities are found for homogeneous plasma.

$$\begin{aligned}
\bar{n}_i &= \frac{\omega_{pi}^2}{4\pi e} \left[\frac{-K_{\perp}^2 \phi}{\Omega_i^2} + \frac{K_{\parallel}^2}{\omega_i^2} \psi \right] \left[\frac{2k-1}{2k} + \xi_i Z(\xi_i) \right] \\
\bar{n}_e &= \frac{\omega_{pe}^2}{4\pi e V_{Te}^2} \left[\left(1 - \frac{1}{2k} \right) + \xi Z_k(\xi) \right] \psi \\
\bar{n}_{2d} &= \frac{\omega_{pd}^2}{2de4\pi} \left[\frac{-k_{\perp}^2}{\Omega_d^2} + \frac{k_{\parallel}^2 \psi}{\omega_d^2} \right] \left[\left(\frac{2k-1}{2k} \right) + \xi_d Z(\xi_d) \right] \dots\dots\dots(3)
\end{aligned}$$

It is observed that essential feature of the kinetic Alfvén wave is retained even in this ideal case. For monwell's equation we use the quasi-neutrality condition (14)

$$\tilde{n}_i = \tilde{n}_e + Z_d \bar{n}_d$$

We get relation between ψ and ϕ as:

$$= n_e + 2d\bar{n}_d$$

$$\phi = \frac{\Omega_d^2}{k_{\perp}^2} \left[\frac{\omega_{pc}^2}{\omega_{pd}^2 v_{T\parallel}^2 \left(\frac{2k-1}{2k} \right)} - \frac{k_{\parallel}^2}{\omega^2} \left(1 + \frac{SP}{Q} \right) R^{-1} \psi \right] \dots\dots(4)$$

Where

$$P = \left[\frac{2k_i-1}{2k_i} \right], \quad Q = \left[\frac{2k_d-1}{2k_d} \right],$$

$$S = \frac{N_0}{N_d} \frac{m_d}{m_i} \frac{1}{z_d^2}, \quad R = \frac{SP}{Q} \frac{\Omega_d^2}{\Omega_i^2}$$

Using perturbed ion, electron and dust particle densities n_i , n_e and n_d and Ampere's law in the parallel direction, we obtained the equation:

$$\frac{\partial}{\partial z} \nabla_{\perp}^2 (\phi - \psi) = \frac{4\pi}{c^2} \frac{\partial}{\partial t} J_z \dots\dots\dots (5)$$

$$\text{Where } J_z = c \int_0^{\infty} 2kV_{\perp} dV_{\perp} \int_{-\infty}^{\infty} dV_{\parallel} [f(V)u_z(r, t) + V_{\parallel}n_1(r, t)_i - f(V)u_z(r, t) + V_{\parallel}n_1(r, t)_e]$$

With the help eq. (4) and (5), one can obtain the dispersion relation for the kinetic Alfvén wave in dusty plasma as:

$$\begin{aligned}
\left(1 - \frac{\omega^2}{K_{\parallel}^2 C_d^2 Q V_{Te}^2} \right) \left(1 - \frac{\omega^2}{V_A^2 k_{\parallel}^2} R \right) &= \frac{k_{\perp}^2 \omega^2 \Omega}{k_{\parallel}^2 \Omega_d^2} - \frac{\omega_{pi}^2 \omega_{pc}^2 \omega^2 p}{\omega_{pd}^2 c^2 K_{\parallel}^2 \Omega_i^2 Q V_{Te}^2} \left(\frac{T_{\parallel i}}{m_i} \right) + \frac{\omega_{pi}^2 P}{c^2 \Omega_i^2} \left(\frac{T_{\parallel i}}{m_i} \right) \left(1 + \frac{SP}{Q} \right) - \frac{\omega_{pi}^2 \omega^2 p k_{\perp}^2}{c^2 K_{\parallel}^2 \Omega_i^2 \Omega_d^2} \left(\frac{T_{\parallel i}}{m_i} \right) + \\
\frac{\omega_{pi}^2 \omega^2 PR}{c^2 K_{\parallel}^2 \Omega_d^2} + \frac{\omega_{pc}^2 \omega^2}{c^2 \Omega_d^2 V_{Te}^2 k_{\parallel}^2} \left(\frac{T_{\parallel d}}{m_d} \right) &- \frac{\omega_{pd}^2}{c^2 \Omega_d^2} \left(\frac{T_{\parallel d}}{m_d} \right) - \frac{\omega_{pd}^2 Q}{c^2 \Omega_d^2} \left(\frac{T_{\parallel d}}{m_d} \right) + \frac{\omega_{pd}^2 Q}{c^2 \Omega_d^2} \left(\frac{T_{\parallel d}}{m_d} \right) \frac{k_{\perp}^2 \omega^2 R}{k_{\parallel}^2 \Omega_d^2} - \frac{SP}{Q}
\end{aligned}$$

$$\text{Where } C_d^2 = \frac{\omega_{pd}^2 V_{Te}^2}{\omega_{pc}^2},$$

$$V_A^2 = \frac{c^2 \Omega_i^2}{\omega_{pd}^2}$$

is the square of dust acoustic speed and V_A is the square of alfvén's speed.

$$\frac{\gamma}{\omega} = \frac{\pi^{1/2} \omega}{k_{\parallel} V_{Te} \left[1 + \frac{\omega_{pi}^2 k_{\parallel}^2 T_{\parallel} P}{\omega^2 \omega_{pe}^2 m_e} + \frac{\omega_{pd}^2 k_{\parallel}^2 T_{\parallel} Q}{\omega^2 \omega_{pe}^2 m_e} \times \left(1 + \frac{\omega^2}{k_{\parallel}^2 v_{Te}^2} \right)^{-(k+1)} \right]}$$

CONCLUSION

Based on the characteristics of plasma in the auroral region, we consider density gradient and kappa distribution function and adopt the particle aspect analysis to investigate the exciting kinetic Alfvén wave in this paper.

The velocity kappa distribution function is an important factor for kinetic Alfvén wave excitation, especially in auroral region .

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