Dust Ion Acoustic Solitary Wave in Weakly Relativistic Dusty Plasma with Non-thermal Ions

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Abstract. Characteristics ofthe nonlinear dust ion acoustic solitary waves in dusty plasma with weak relativistic effect are examined. We consider the weakly relativistic effect of electron species, the ion species with nonthermal distribution and charged dust grains. The nonlinear Korteweg-de Vries equation is derived from the governing normalized basic set of equations. The numerical calculations show the influence of non-thermal ion population on the width and amplitude of dust ion acoustic solitary waves. Space plasma and interstellar medium are the areas where the results of the work can be applied.

INTRODUCTION

The suspension of dust particles in normalelectron-ion plasma has introduced the more complexity in the plasma system and so it named as "dusty plasma". Its presence is abundant in interplanetary space, circumsolar, interstellar clouds, planetary nebulas, cometary tails etc. [1-2]. The extensive work has been done to analyze the linear andnonlinear wave propagation in dusty plasma. Singh and Rao [3] have studied the propagation of linear and nonlinear dust acoustic waves in dusty plasma. Dorranian and Sabetkar [4] have carried out the investigation in dusty plasma and studied the dust acoustic solitary waves considering two non-thermal ion species. The investigation on nonlinear ion acoustic solitary wave in magnetized electron positron ion plasmas has been carried out by Mahmood et al. [5].Recently, the dust ion acoustic solitary structures in collisionless dusty plasma considering isothermal positrons have been studied by Paul et al. [6] using Sagdeev potential technique.

However, all the above mentioned works have been constrained in the nonrelativistic plasma. But in some situations such as in the Van Allen radiation belts, in interstellar medium, space plasma, in earth magnetosphere, where the velocity of electron species becomes approximately similar to the speed of light, the consideration of relativistic behavior in plasma shows significant role in modifying the nonlinear structures.In this direction, Singh and Dahiya [7] have obtained the Korteweg-de Vries (KdV) equation to study the influence of ion temperature and density of plasma on the behavior of ion acoustic solitons in relativistic plasma. The behavior of spherical and cylindrical dust ion acoustic solitary waves in dusty plasma with the presence of relativistic ions has been studied by Liu et al. [8]. The dust ion acoustic solitary waves in multicomponent plasma considering the weak relativistic effects of electron and ion have studied byKalita and Choudhary [9]. Moreover, several plasma workers dealt with the dusty plasma incorporating the nonthermal ion population to study the nonlinear waves and structures. Zhang and Wang [10] have reported the influence of two temperature nonthermal ions on dust acoustic solitary waves. The KdV- Burgers equation is obtained by Shahmansouri and Mamun [11] to establish the effect of nonthermal ion and electrons on the dust acoustic shock waves in magnetized viscous dusty plasma. Thus, the non-thermal ion distribution is very important to study the non-linear waves and structures in dusty plasma. In the present investigation, following the work of Kalita and Choudhary [9], weak relativistic effect of electrons and nonthermal distribution of ions are considered to observe the role of nonthermal ions on dust ion acoustic solitary waves in dusty plasma accounting weak relativistic electron species.

BASIC EQUATIONS

We have considered homogeneous unmagnetized dusty plasma to study the effect of non-thermal ion population on solitary waves in presence of weakly relativistic electrons. The number density $n_i$of non-thermally distributed ions can be written as

$n_i = \left[1+\beta\psi\delta+\beta\psi^2\delta^2\right]\exp(-\psi\delta)$ (1)

The symbol $\beta = 4\alpha/(1+3\alpha)$ where α represents the non-thermal ion populationand $\delta = T_{e0}/T_{i0}$. The electrostatic potential, ion temperature, electric charge and Boltzmann constant are given by ψ , T_i, e and k_B respectively.

Further the electrons are assumed weakly relativistic as its mass (m_e) increased with its velocity $\boldsymbol{u}_e$. Hence, the continuity and momentum equations forelectron are

$\frac{\partial}{\partial t}(\gamma_e n_e)+\frac{\partial}{\partial z}\cdot(\gamma_e n_e \boldsymbol{u}_e)=0$ (2), $\left(\frac{\partial}{\partial t}+\boldsymbol{u}_e\frac{\partial}{\partial z}\right)(\gamma_e\boldsymbol{u}_e)-\frac{m_d}{m_e}\frac{d\psi}{dz}+\frac{m_d}{\gamma_e m_e n_e}\frac{\partial}{\partial z}(\gamma_e n_e)=0$ (3)

where $\gamma_e = (1-u_e^2/c^2)^{-1/2}$, n_e, T_e and $p_e(=n_e k_B T_e)$ represent relativistic factor, number density, temperature and pressure of electron respectively and c represents speed of light.

The dust is assumed nonrelativistic and its dynamics can be represented as

$\frac{\partial n_d}{\partial t}+\frac{\partial}{\partial z}\cdot(n_d\boldsymbol{u}_d)=0$ (4), $\left(\frac{\partial}{\partial t}+\boldsymbol{u}_d\frac{\partial}{\partial z}\right)\boldsymbol{u}_d - Z_d\frac{\partial\psi}{\partial z}=0$ (5)

The mass, velocity, number density, charge and temperature of negatively charged dust grains is symbolized by m_d, $\boldsymbol{u}_d$, n_d, q_d and T_d respectively.

The Poisson's equation is expressed as

$\frac{\partial^2\psi}{\partial z^2} = -(n_i - \gamma_e n_e - Z_d n_d)$ (6)

Equations(1)-(6) are normalized using the following dimensionless quantities as

$$n_{d,e,i} = \frac{n_{d,e,i}}{n_{i0}},\ \frac{\partial}{\partial z} = \frac{\partial}{\partial z}\left(\frac{k_B T_e}{4\pi e^2 n_{i0}}\right)^{1/2},\ \psi = \frac{e\psi}{k_B T_e},\ \boldsymbol{u}_{d,e,i} = \boldsymbol{u}_{d,e,i}\left(\frac{m_d}{k_B T_e}\right)^{1/2},\ \frac{\partial}{\partial t} = \frac{\partial}{\partial t}\left(\frac{m_d}{4\pi e^2 n_{i0}}\right)^{1/2}$$

DUST ION ACOUSTIC SOLITARY WAVES

In order to study the impact of non-thermal ion population onan ion acoustic solitary wave in weakly relativistic dusty plasma the reductive perturbation method has been used. The stretched coordinates are introduced as $\xi = \varepsilon^{1/2}(z-\lambda t)$ and $\tau = \varepsilon^{3/2} t$ where ε and λ represents the nonlinearity measurable parameter and phase velocity respectively.

The plasma variables n, $\boldsymbol{u}$and ψ are expanded in the power series of ε in the following form as

$$\begin{pmatrix} n_e \\ n_i \\ n_d \\ \psi \\ u_e \\ u_i \\ u_d \end{pmatrix} = \begin{pmatrix} n_{e0} \\ 1 \\ n_{d0} \\ 0 \\ u_{e0} \\ 0 \\ 0 \end{pmatrix} + \varepsilon \begin{pmatrix} n_{e1} \\ n_{i1} \\ n_{d1} \\ \psi_1 \\ u_{e1} \\ u_{i1} \\ u_{d1} \end{pmatrix} + \varepsilon^2 \begin{pmatrix} n_{e2} \\ n_{i2} \\ n_{d2} \\ \psi_2 \\ u_{e2} \\ u_{i2} \\ u_{d2} \end{pmatrix} + \dots\dots\dots \quad (7)$$

Now, using the stretched coordinates and applying the above expansion (7)inequations (1)-(6) and collecting the lowest order terms of ε from nonthermal distribution of ion, continuity and momentum equations of electron and dust as well as from Poisson equation of electrostatic potential, following results are obtained

$n_{i1} = (\beta - 1)\delta\psi_1$ (8), $u_{e1} = \dfrac{\chi\psi_1}{M\chi^2(1+1.5u_{e0}^2/c^2)-1}$ (9) $\quad n_{e1} = \dfrac{-\{1+1.5(u_{e0}^2/c^2)-(\lambda u_{e0}/c^2)\}n_{e0}\psi_1}{\{M\chi^2(1+1.5u_{e0}^2/c^2)-1\}(1+0.5u_{e0}^2/c^2)}$ (10),

$u_{d1} = \dfrac{-z_d\psi_1}{\lambda}$ (11), $n_{d1} = \dfrac{-z_d n_{d0}\psi_1}{\lambda^2}$ (12), $n_{e1}\left(1+\dfrac{u_{e0}^2}{2c^2}\right)+u_{e1}\left(\dfrac{n_{e0}u_{e0}}{c^2}\right)-n_{i1}+z_d n_{d1}=0$ (13)

where $\chi = (-\lambda + u_{e0})$

Now,again collecting the next higher order of ε from nonthermal distribution of ion, we have

$n_{i2} = (\beta-1)\delta\psi_2 + \dfrac{\delta^2}{2}\psi_1^2$ (14),

and on collecting the continuity and momentum equations of electron and dust which includes next higher order terms ($\sim\varepsilon^{5/2}$) we obtained

$$\left(\frac{n_{e0}u_{e0}}{c^2}\right)\frac{\partial u_{e1}}{\partial\tau}+\left(1+\frac{1}{2}\frac{u_{e0}^2}{c^2}\right)\frac{\partial n_{e1}}{\partial\tau}+\frac{n_{e0}}{c^2}(-\lambda+3u_{e0})u_{e1}\frac{\partial u_{e1}}{\partial\xi}+\left(1+\frac{3}{2}\frac{u_{e0}^2}{c^2}-\frac{\lambda u_{e0}}{c^2}\right)\frac{\partial n_{e1}u_{e1}}{\partial\xi}$$
$$+n_{e0}\left(1+\frac{3}{2}\frac{u_{e0}^2}{c^2}-\frac{\lambda u_{e0}}{c^2}\right)\frac{\partial u_{e2}}{\partial\xi}+\chi\left(1+\frac{1}{2}\frac{u_{e0}^2}{c^2}\right)\frac{\partial n_{e2}}{\partial\xi}=0 \quad (15)$$

$$n_{e0}\left(1+\frac{3}{2}\frac{u_{e0}^2}{c^2}\right)\left(1+\frac{1}{2}\frac{u_{e0}^2}{c^2}\right)\frac{\partial u_{e1}}{\partial\tau}++\chi\left(1+\frac{3}{2}\frac{u_{e0}^2}{c^2}\right)\left(1+\frac{1}{2}\frac{u_{e0}^2}{c^2}\right)n_{e1}\frac{\partial u_{e1}}{\partial\xi}+\left(\frac{1}{M}\frac{u_{e0}}{c^2}\right)\frac{\partial u_{e1}n_{e1}}{\partial\xi}+n_{e0}\left\{\frac{1}{Mc^2}+\frac{4\chi u_{e0}}{c^2}\right.$$
$$\left.\left(1+\frac{1}{2}\frac{u_{e0}^2}{c^2}\right)+\left(1+\frac{3}{2}\frac{u_{e0}^2}{c^2}\right)\left(1+\frac{1}{2}\frac{u_{e0}^2}{c^2}\right)+\frac{\chi u_{e0}}{c^2}\left(1+\frac{3}{2}\frac{u_{e0}^2}{c^2}\right)\right\}u_{e1}\frac{\partial u_{e1}}{\partial\xi}-\left(1+\frac{1}{2}\frac{u_{e0}^2}{c^2}\right)\frac{n_{e1}}{M}\frac{\partial\psi_1}{\partial\xi}-\left(\frac{1}{M}\frac{n_{e0}u_{e0}}{c^2}\right)u_{e1}\frac{\partial\psi_1}{\partial\xi} \quad (16)$$
$$+\frac{1}{M}\left(1+\frac{1}{2}\frac{u_{e0}^2}{c^2}\right)\frac{\partial n_{e2}}{\partial\xi}-\frac{n_{e0}}{M}\left(1+\frac{1}{2}\frac{u_{e0}^2}{c^2}\right)\frac{\partial\psi_2}{\partial\xi}+n_{e0}\left\{\chi\left(1+\frac{3}{2}\frac{u_{e0}^2}{c^2}\right)\left(1+\frac{1}{2}\frac{u_{e0}^2}{c^2}\right)+\frac{1}{M}\frac{u_{e0}}{c^2}\right\}\frac{\partial u_{e2}}{\partial\xi}=0$$

$$\frac{\partial n_{d1}}{\partial\tau}+\frac{\partial n_{d1}u_{d1}}{\partial\xi}-\lambda\frac{\partial n_{d2}}{\partial\xi}+\frac{n_{d0}\partial u_{d2}}{\partial\xi}=0 \quad (17)$$

$$\frac{\partial u_{d1}}{\partial\tau}+u_{d1}\frac{\partial u_{d1}}{\partial\xi}-\lambda\frac{\partial u_{d2}}{\partial\xi}-z_d\frac{\partial\psi_2}{\partial\xi}=0 \quad (18)$$

Similarlycollecting the next higher order terms ($\sim\varepsilon^2$) from Poisson equation of electrostatic potential

$$\frac{\partial^2\psi_1}{\partial\xi^2}-\left(1+\frac{u_{e0}^2}{2c^2}\right)n_{e2}-\frac{n_{e0}u_{e0}u_{e2}}{c^2}+(\beta-1)\delta\psi_2+\frac{\delta^2\psi_1^2}{2}-z_d n_{d2}-\frac{\chi\psi_1^2}{M\chi^2(1+1.5u_{e0}^2/c^2)-1}\left(\frac{\chi}{2c^2}-\frac{u_{e0}\{1+1.5u_{e0}^2/c^2-\lambda u_{e0}/c^2\}}{(1+0.5u_{e0}^2/c^2)c^2}\right)=0 \quad (19)$$

Now substituting the require value of n_{e2}, u_{e2} and n_{d2} from above equations (14)– (18) in equation (19), we obtained the KdV form of equation

$$\frac{\partial\psi_1}{\partial\tau}+A\psi_1\frac{\partial\psi_1}{\partial\xi}+B\frac{\partial^3\psi_1}{\partial\xi^3}=0 \quad (20),$$

where

$$A=\frac{1}{B}\left[\left(1+\frac{2u_{e0}^2}{c^2}\right)+M\left\{3\chi^2+\frac{2\lambda^4}{c^2}+\frac{8u_{e0}^4}{c^2}+\frac{18\lambda^2u_{e0}^2}{c^2}-\frac{8\lambda^3u_{e0}}{c^2}-\frac{32\lambda u_{e0}^3}{c^2}+\frac{12\lambda u_{e0}^4}{c^2}\right\}+\frac{z_d(\beta-1)\delta}{\lambda^2}-\frac{2z_d^3n_{d0}}{\lambda^4}+\right.$$
$$\left.\frac{(z_d-z_d^2n_{d0})/\lambda^2}{M\chi^2(1+1.5u_{e0}^2/c^2)-1}\right],\ B=\left[\frac{-2M\chi(1-z_dn_{d0})(1+2u_{e0}^2/c^2)}{(1+1.5u_{e0}^2/c^2)\{M\chi^2(1+1.5u_{e0}^2/c^2)-1\}^2}+\frac{z_d^2n_{d0}}{\lambda^2}-\frac{(1-z_dn_{d0})/\lambda}{\{M\chi^2(1+1.5u_{e0}^2/c^2)-1\}}-\frac{(\beta-1)\delta}{\lambda}\right]^{-1}$$

The stationary wave solution can be find by transforming space and time coordinates using $\eta=\xi-U_0\tau$, in which U_0 represents the constant speed of solitary wave and boundary conditions $\psi_1=0,\ \partial\psi_1/\partial\eta=0,\ \partial^2\psi_1/\partial\eta^2=0$ for

$\eta \to \infty$. Thus, the obtained solution of Eq. (20) can be given as $\psi_1 = \psi_0 \sec h^2(\eta/\Delta)$, where $\psi_0 (= 3U_0/A)$ and $\Delta = (4B/U_0)^{1/2}$.

In order to see the effect of nonthermal ion numerically on the solitary waves, Fig. 1 is plotted. The variation of electrostatic potential of dust ion acoustic solitary waves for three values of nonthermal ion is shown in Fig. 1. The constant values of other parameters are $u_{e0} = 10$, $z_d = 10$, $c = 500$, $M = 0.01$, and $\delta = 0.1$. It is seen that the soliton amplitude and width increases with nonthermal ion for weakly relativistic dusty plasma.

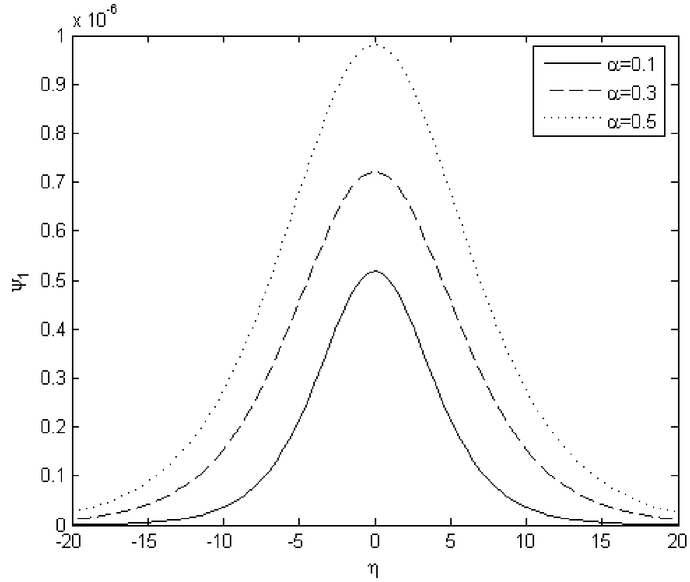

FIGURE 1. Variation of electrostatic potential of solitary waves for three different values of nonthermal ion

CONCLUSIONS

In the considered model, the effect of non-thermal ion on dust acoustic solitary waves has been studied. It can be concluded that dust ion acoustic solitary waves significantly affected by the nonthermal ion population in dusty plasma with weakly relativistic electrons.

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