Low Frequency Modes of Propagation in Strongly Coupled Degenerate Dusty Plasma

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Abstract. In the present work, the dust acoustic modes of strongly coupled quantum plasma have been examined considering the dust polarization force with degenerate electron and ion species. We have incorporated the statistical and diffraction effects of all the three plasma species i.e. electron, ion and dust grains. The normal mode analysis technique is used to obtain the general dispersion relation. Further, the general dispersion relation is discussed analytically as well as numerically for hydrodynamic and kinetic regimes. The numerical results for the strongly coupled degenerate plasma have been shown to see the effect of polarization force on dust acoustic modes.

INTRODUCTION

In the last few years, dusty (complex) plasmas have gained a remarkable interest of several investigators due to their wide applications in astrophysical and space plasmas as well as in laboratory plasmas. Dusty plasma characterizes an ionized gas consist of negatively charged electrons, positive ions, neutrals and sub-micron sized dust particulates. When the charged dust particles combined into an electron ion plasma, the existing conventional wavemodes altered and also some new types of low frequency wave modes arises in the dusty plasma which generate instabilities [1-4]. In a dusty plasma, different forces such as polarization force [5, 6], neutral and ion drag forces [7], pressure force [5], charge gradient force [8, 9], gravitational force [10] etc. acted upon the dust grains. In a non-uniform plasma, the constant charged dust grains are acted by the polarization force which impulses the dust particles into the smaller Debye length region. In this regard, a lot of theoretical work on polarization force has been done by many researchers. Recently, Kharapaket al. [9] studied the effect of polarization force on low frequency dusty modes. The effect of polarization force on Jeans instability using generalized hydrodynamic model has been discussed by Sharma [11] in a strongly coupled magneto-dusty plasma.

However, the plasma is also found in strong coupling state in various laboratory devices and astrophysical and space plasmas. In strongly coupled dusty plasmas, the Coulomb coupling energy of the dust grains dominate over the thermal energy and the coupling parameter of dust grains is greater than unity [11].Therefore, out of the three plasma species the dust grains are present in strongly coupled degenerate state and the remaining two species i.e. electrons and ions are exist in weakly coupled degenerate state in the present work. In this work, we mainly focus on the low frequency dust modes in the strongly coupled degenerate plasma incorporating the dust polarization force in the light of the consequences given by Sharma and Jain [4] and Shukla and Ali [12]. None of the works has investigated the different dust modes in strongly coupled plasma with polarization force and quantum effects. Hence, in the present work, the low frequency dust modes in the strongly coupled plasma have been investigated.

LINEARIZED MODEL EQUATIONS

Let us consider a strongly coupled quantum dusty plasma system consisting of degenerate electrons, ions and charged dust grains. The electrons and ions are assumed to be immobile and inertialesswith respect to the dust grains. When the temperature of the system is very low, the de Broglie wavelength of the plasma species (i.e. electron, ion and dust particulates) becomes larger than the system dimensions and therefore it is necessary to include the quantum mechanical effects in the dynamics of the plasma system. Thus, in the present system, the quantum mechanical effects in the form of statistical and diffraction terms have been incorporated in the equations of motion of electron, ion and dust species separately.Therefore, in such a strongly coupled quantum dusty plasma system the dynamics of the plasma species,can be governed by the following set of linearized perturbed equations:

$$0 = e\nabla\psi_1 - \frac{2k_B T_{Fe}}{n_{e0}}\nabla n_{e1} + \frac{\hbar^2}{4m_e n_{e0}}\nabla^3 n_{e1}, \quad (1)$$

$$0 = -e\nabla\psi_1 - \frac{2k_B T_{Fi}}{n_{i0}}\nabla n_{i1} + \frac{\hbar^2}{4m_i n_{i0}}\nabla^3 n_{i1}, \quad (2)$$

where $e, K_B, \psi_1, T_{Fe,i}, n_{e,i1}, n_{e,i0}, m_{e,i}$ and $\hbar$ are the electron charge, Boltzmann's constant, electrostatic potential, Fermi temperature of electron and ion, perturbed number density of electron and ion, equilibrium electron and ion number density, mass of electron and ion and Planck's constant (divided by 2π) respectively. In Eqs. (1) and (2) the second term on the right hand side shows the quantum statistical effect of negative electrons and positive ions respectively while the third term represents the diffraction due to the Bohm potential.

We use the generalized hydrodynamic (GH) model to describe the negatively charged dust grains which is depicted by the momentum transfer and continuity equations.

$$\frac{\partial}{\partial t}n_{d1} + n_{d0}\nabla \boldsymbol{v}_{d1} = 0, \quad (3)$$

$$\left(1+\tau_m\frac{\partial}{\partial t}\right)\left[m_d n_{d0}\left(\frac{\partial}{\partial t}+\upsilon_{dn}\right)\boldsymbol{v}_{d1} - Z_d e n_{d0}\nabla(1-R)\psi_1 + 2k_B T_{Fd}\nabla n_{d1} - \frac{\hbar^2}{4m_d}\nabla^3 n_{d1}\right] = \eta_l \nabla^2 \boldsymbol{v}_{d1} \quad (4)$$

In Eqs. (3) and (4) the symbols $v_{d1}, n_{d0}, n_{d1}, m_d, Z_d, \upsilon_{dn}, R, \tau_m, T_{Fd}$ and η_l represent the perturbed dust velocity, equilibrium dust number density, perturbed number density, dust mass, number of charges on dust grains, dust neutral collision frequency, polarization parameter, relaxation time, dust Fermi temperature and viscoelastic parameter respectively.

The first order perturbed charge neutrality condition is

$$n_{i1} = n_{e1} + Z_d n_{d1} \quad (5)$$

DISPERSION RELATION AND DISCUSSION

We assume that all the first order perturbed parameters of Eqs. (1) – (4) are proportional to $\exp i(\boldsymbol{k}.\boldsymbol{r} - \omega t)$ in which ω and $\boldsymbol{k}$ are the perturbation frequency and wave number respectively. Further, employing this perturbation in Eqs. (1) – (4) we get the perturbed densities for plasma species (ion, electron and dust) and using these perturbed densities in charge neutrality condition (5) we found the general dispersion relation for strongly coupled degenerate dusty plasma as

$$(1-i\omega\tau_m)\left\{(-i\omega+\upsilon_{dn})i\omega - k^2C_{Fd}^2(1+\delta_d)\right\} + \frac{\eta_l k^2 i\omega}{m_d n_{d0}} - \frac{k^2C_{dq}^2(1-R)(1-i\omega\tau_m)(1+\delta_i)}{1+\zeta} = 0 \quad (6)$$

where we have used $C_{dq} = Z_d\left(2k_B T_{Fi} n_{d0}/m_d n_{i0}\right)^{1/2}$, $C_{Fd}^2 = 2k_B T_{Fd}/m_d$ and $\zeta = (1+\delta_i)n_{e0}T_{Fi}/n_{i0}T_{Fe}(1+\delta_e)$ in which $\delta_{e,i} = \hbar^2 k^2/8m_{e,i}k_B T_{Fe,i}$ and $\delta_d = \hbar^2 k^2/8m_d k_B T_{Fd}$.

The above general dispersion relation describes the influences of polarization force on low frequency dust modes in strongly coupled dusty degenerate plasma. In absence of strong coupling of dust this relation resembles with the dispersion relation (8) of Sharma and Jain []. Thus, this work is the advancement of the work given by Sharma and Jain []. Further, the general dispersion relation given by Eq. (6) can be explained in two regimes as hydrodynamic and kinetic regimes.

In the hydrodynamic limit i.e. $\omega << 1/\tau_m$ the low frequency dusty mode given by Eq. (6) takes the shape as (using $-i\omega = \sigma$)

$$\sigma^2 + \sigma\left\{\upsilon_{dn} + V_{cd}^2 k^2 \tau_m\right\} + k^2 C_{Fd}^2\left(1+\delta_d\right) + \frac{k^2 C_{dq}^2 (1-R)(1+\delta_i)}{1+\zeta} = 0 \quad (7)$$

where $V_{cd}^2 = \eta_l / m_d n_{d0} \tau_m$. The dispersion relation (7) shows that the polarization force and collision frequency significantly modified expression of strongly coupled degenerate dusty plasma system. Eq. (7) shows that the dust polarization force along with the neutral particles dissipating the dust mode.

In the kinetic limit i.e. $\omega >> 1/\tau_m$ the expression given by Eq. (6) with $-i\omega = \sigma$ becomes as

$$\sigma^2 + \sigma\upsilon_{dn} + V_{cd}^2 k^2 + k^2 C_{Fd}^2\left(1+\delta_d\right) + \frac{k^2 C_{dq}^2 (1-R)(1+\delta_i)}{1+\zeta} = 0 \quad (8)$$

The solution of above equation can be written as

$$\sigma = -\frac{\upsilon_{dn}}{2} \pm \left[\frac{\upsilon_{dn}^2}{4} - V_{cd}^2 k^2 \tau_m - k^2 C_{Fd}^2\left(1+\delta_d\right) - \frac{k^2 C_{dq}^2 (1-R)(1+\delta_i)}{1+\zeta}\right]^{1/2} \quad (9)$$

Equation (9) with $\upsilon_{dn} << \sigma$, becomes as

$$\sigma = k\left[-V_{cd}^2 \tau_m - C_{Fd}^2\left(1+\delta_d\right) - \frac{C_{dq}^2 (1-R)(1+\delta_i)}{1+\zeta}\right]^{1/2} \quad (10)$$

The above equation gives the dispersion properties of dust acoustic wave in the strongly coupled quantum plasma. It is clear from the solution that low frequency dust modes are modified due to the polarization force and relaxation time.

NUMERICAL ESTIMATION

In order to study the dispersion characteristics of low frequency degenerate dust modes numerically in strongly coupled plasma we normalize the dispersion relations (7) and (8) with the dust plasma frequency considering $\upsilon_{dn} << \sigma$. In the normalization process, we have used the some dimensionless parameters which are

$$\sigma^* = \frac{\sigma}{\omega_{pd}}, V_{cd}^* = \frac{V_{cd}}{c_d}, \tau_m^* = \tau_m \omega_{pd}, k^* = \frac{k c_d}{\omega_{pd}}, \ \delta_{Te} = \frac{T_{Fd}}{T_{Fe}}, \delta_{Ti} = \frac{T_{Fd}}{T_{Fi}}, \ \delta_{me} = \frac{m_d}{m_e}, \delta_{mi} = \frac{m_d}{m_i}, H_d = \frac{\hbar\omega_{pd}}{2k_B T_{Fd}}, \ \delta_1 = \frac{n_{i0} T_{e0}}{n_{e0} T_{i0}}$$

Using these dimensionless parameters in Eq. (7) and (8), we get

$$\sigma^{*2} + \sigma^* V_{cd}^{*2} k^{*2} \tau_m^* + k^{*2}\delta_{Ti}\left(1 + \frac{H_d^2}{4}\delta_{Ti} k^{*2}\right) + \frac{k^{*2}\dfrac{Z_{d0} n_{d0}}{n_{i0}}(1-R)\left(1 + \dfrac{H_d^2}{4}\delta_{mi}\delta_{Ti}^2 k^{*2}\right)}{\left(1 + \delta_1\left\{1 + \dfrac{H_d^2}{4}\delta_{mi}\delta_{Ti}^2 k^{*2}\right\}\left\{1 + \dfrac{H_d^2}{4}\delta_{me}\delta_{Ti}\delta_{Te} k^{*2}\right\}^{-1}\right)} = 0 \quad (11)$$

andin the kinetic regime

$$\sigma^{*2} + V_{cd}^{*2} k^{*2} + k^{*2}\delta_{Ti}\left(1 + \frac{H_d^2}{4}\delta_{Ti} k^{*2}\right) + \frac{k^{*2}\dfrac{Z_{d0} n_{d0}}{n_{i0}}(1-R)\left(1 + \dfrac{H_d^2}{4}\delta_{mi}\delta_{Ti}^2 k^{*2}\right)}{\left(1 + \delta_1\left\{1 + \dfrac{H_d^2}{4}\delta_{mi}\delta_{Ti}^2 k^{*2}\right\}\left\{1 + \dfrac{H_d^2}{4}\delta_{me}\delta_{Ti}\delta_{Te} k^{*2}\right\}^{-1}\right)} = 0 \quad (12)$$

Numerical calculations of roots of Eqs. (11) and (12) have been done and shown in the form of Figs. (1) and (2). In Fig. 1 the effect of dust polarization force on low frequency dust modes has been shown in the hydrodynamic regime of strongly coupled dusty plasma. It is clear from Fig. (1) that the presence of polarization parameter decreases the normalized frequency with normalized wave number. Figure (2) shows the effect of strong correlation on dust modes in kinetic regime of strongly coupled dusty plasma. From Fig. (2) we find that the strong correlation increases the dispersion frequency with the wave number. Hence the dust polarization parameter destabilizes whereas the strong correlation stabilizes the present system.

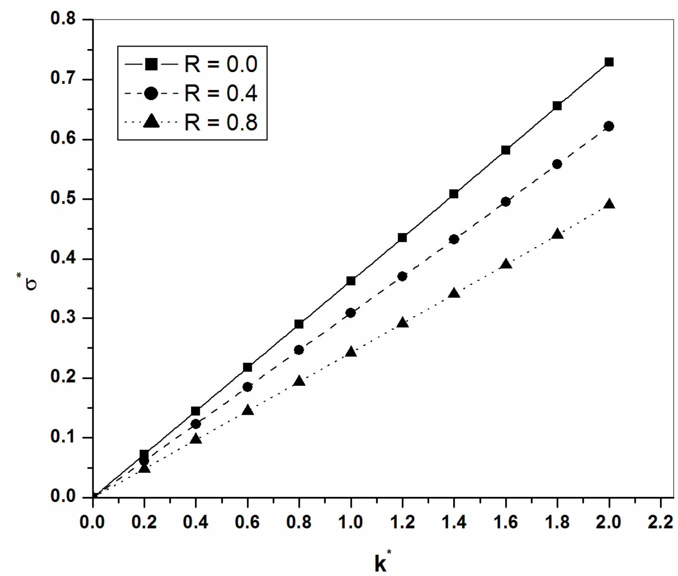

FIGURE (1): The normalized wave frequency σ^* versus normalized wave number k^* for different values of dust polarization parameter.

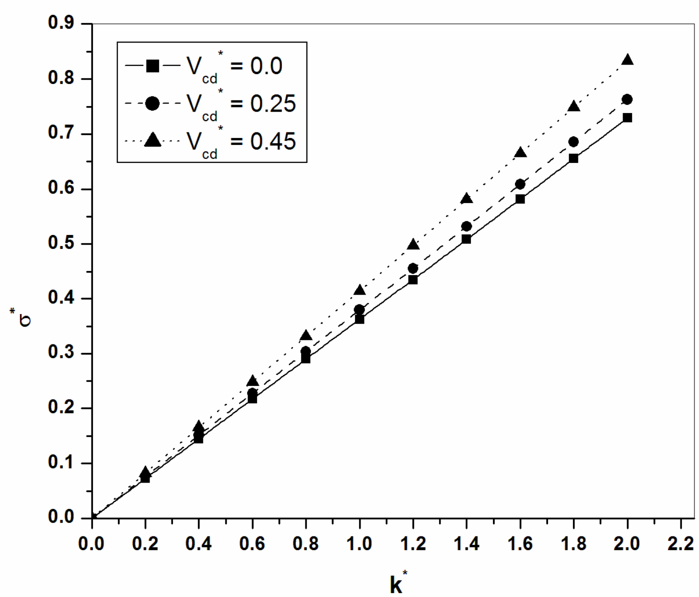

FIGURE (2): The normalized wave frequency σ^* versus normalized wave number k^* for different values of strong coupling parameter.

CONCLUSIONS

In this work, we have studied the low frequency dust modes in degenerate strongly coupled dusty plasma using GH model. The dusty modes are examined in the hydrodynamic and kinetic regimes. It is noted analytically as well as numerically that the dust polarization force destabilize the present system and the strong correlation effect stabilize the system. The results found from the present work are useful in astrophysical and laboratory plasmas where strong coupling of plasma species are present.

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