Influence of Strong Correlation and Polarization Force of Dust on Propagation Modes of Degenerate Dusty Plasma

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Abstract. The effects of dust polarization force and charge gradient have been investigated on low frequency strongly coupled dust acoustic modes. We have taken the massive dust particulates as polarized and degenerate electrons and distribution of ions are assumed as  Maxwellian. The general dispersion relation has been obtained by employing the normal mode method and generalized hydrodynamic model. The obtained general dispersion relation is further studied for a characteristic timescale which gives two different frequency regimes hydrodynamic and kinetic regimes. The numerical results have been presented by plotting the graphs between frequency and wave number which show the effect of different parameters on dust acoustic modes.

INTRODUCTION

Now-a-days, the dusty plasma (DP) has attained a great interest due to its vital role in describing the space and astrophysical regions as well as in the laboratory devices. The first theoretical model of DP has been predicted by Rao et al. [1]. Since the dust particulates are highly positive or negative charged therefore due to the mutual Coulomb interaction they are strongly attached with each other and form a strongly coupled state. In recent years, a great deal of attention has been given in the study of different types of wave modes and instabilities in strongly coupled dusty plasmas using the generalized hydrodynamic (GH) model [2, 3].The dust acoustic waves have been studied by Shukla and Mamun [4] using GH model and KdV equations. Mamun et al. [5] have discussed the low-frequency dust waves with dust charge variation in strongly coupled dusty plasma.

In a strongly coupled dusty plasma model, the charged species are affected by the several forces such as pressure, gravitational, neutral, electron and ion drag forces, polarization, charge gradient forces etc. In case of constant charge and non-uniform plasma, the polarization force acted upon the plasma particles. The dust polarization force has been studied by Hamaguchi and Farouki [6, 7]. But when the dust grain charge is inconstant the charge gradient force comes into existence. This force drives the negatively (positively) charged plasma species in the region of lower (higher) charge. In this regard, the influence of charge gradient force and dust polarization force on the low frequency dust modes have been described by Khrapak et al. [8, 9]. The effect of dust charge gradient along with the dust charge electric field has been discussed by Shukla [10] to study the novel instabilities of dusty plasma. Therefore, looking to the importance of charge gradient and polarization force dusty modes, we have studied the effect of polarization force, charge gradient and strong correlation on dust propagation modes in degenerate dusty plasma.
THEORETICAL MODEL

We consider a generalized hydrodynamic (GH) model to describe the degenerate dusty plasma system. In this plasma model, we take the effects of dust polarization and charge gradient forces in the dust momentum equation with strong coupling. In this system, the ions show the Maxwellian behavior whereas electrons are degenerate and give the quantum effect due to the Bohm potential term. We assume that the perturbed quantities in the present strong coupled dusty plasma system vary as \( \exp(i(\mathbf{k} \cdot \mathbf{r} - \omega t)) \) where \( \mathbf{k} = k_x \hat{x} + k_z \hat{z} \) is the wave number and \( \omega \) is the perturbation frequency. Thus, the expressions for perturbed number densities of Maxwellian ions \( (n_i) \) and degenerate electrons \( (n_e) \) are

\[
\begin{align*}
    n_i &= -\frac{n_i^0 \epsilon \phi_i}{T_i} \quad (1) \\
    0 &= e \nabla \phi_i - \frac{\hbar^2}{4m_i n_i^0 \epsilon} \nabla^2 n_i \quad \text{which gives} \quad n_e = \frac{4m_e n_e^0 \epsilon \phi_i}{\hbar^2 k^2} \quad (2)
\end{align*}
\]

In above Eqs. (1) and (2) \( \phi_i \) is perturbed electrostatic potential, \( n_i^0 \) is equilibrium ion number density and \( n_e^0 \) is equilibrium electron number density, \( T_i \) is ion temperature, \( e \) is the electron charge, \( m_e \) is electron mass and \( h \) is Planck’s constant divided by \( 2\pi \).

The dust charging equation is

\[
\frac{\partial Z_{d\lambda}}{\partial t} + \omega_{\lambda} Z_{d\lambda} = J_0 \left( \frac{n_{d\lambda}}{n_{i0}} - \frac{n_{d\lambda}}{n_{e0}} \right) \quad (3)
\]

where \( Z_{d\lambda} \), \( \omega_{\lambda} \) and \( J_0 \) are perturbed number of charges on dust surface, characteristics charging frequency, ions/electrons equilibrium flux respectively.

The full dynamics of strongly coupled dust is given by the momentum transfer and continuity equations as

\[
\frac{\partial}{\partial t} n_{d\lambda} + n_{i0} \nabla v_{d\lambda} = 0 \quad (4)
\]

\[
\begin{bmatrix}
1 + \tau_w \frac{\partial}{\partial t} \nabla + \nabla_{d\lambda} \right] v_{d\lambda} + \frac{q_{d\lambda} \nabla \phi_i (1 - P)}{m_d} - \frac{q_{d\lambda} \nabla q_{d\lambda}}{\lambda_d m_d} \right] = \eta_i \nabla^2 v_{d\lambda} \quad (5)
\end{bmatrix}
\]

The symbols \( n_{d\lambda} \) and \( n_{d0} \) are perturbed and equilibrium dust number density respectively, \( v_{d\lambda} \) is dust velocity, \( v_{d0} \) is dust neutral collision frequency, \( \tau_w \) is relaxation time, \( q_{d\lambda} = (-eZ_{d\lambda}) \) is equilibrium dust density, \( P \) is polarization parameter, \( \lambda_d \) is Debye length, \( m_d \) is dust mass, and \( \eta_i \) is viscoelastic coefficient.

In Eqs. (3) and (4) using the above mentioned perturbation and after some simple algebra, the perturbed dust number density is obtained as

\[
n_{d\lambda} = -\frac{eZ_{d\lambda} n_{i0} k^2 (1 - i\omega \tau_w)}{m_d \left( 1 - i\omega \tau_w \right)} \tilde{\phi}_i \left[ 1 - P \right] + \frac{J_\lambda \left( \frac{e^2}{T_i} + \frac{4m_e e^2}{\hbar^2 k^2} \right)}{\lambda_e \omega_k} \left[ (-i\omega + \nu_{d\lambda}) i\omega + \frac{\eta_i \omega_k^2 i\omega}{m_d n_{d0}} \right] \quad (6)
\]

The charge neutrality condition for considered system is

\[
n_{i\lambda} - n_{d\lambda} + Z_{d\lambda} n_{i0} + Z_{d\lambda} n_{d\lambda} = 0 \quad (7)
\]
The modes of propagation are discussed deriving the dispersion relation for degenerate dusty plasma system using the value $n_{d1}$, $n_{e1}$, $Z_{d1}$ and $n_{d1}$, thus the obtained dispersion relation is

\[(1 + \sigma \tau_{m}) \left\{ \left( \sigma + u_{d*,m} \right) \sigma + \frac{\eta k^{2} \sigma}{m_{d} n_{d*}} \right\} \left[ 1 + \frac{J_{0} p_{d*}}{\omega_{n_{d*}}} \right] + \frac{H_{qe}}{k^{2}} \left( 1 + \frac{J_{0} p_{e*}}{\omega_{n_{e*}}} \right) = \omega_{p}^{2} k^{2} \left[ (1 - P) + \frac{J_{0} e^{2}}{\lambda_{e} \omega_{e} T_{e0}} + \frac{J_{0} 4m e^{2}}{\lambda_{e} \omega_{e} h^{2} k^{2}} \right] \] (8)

Equation (8) represents the general dispersion relation for viscoelastic degenerate dusty plasma system which shows the influence of charge gradient force, polarization force, dust neutral collision frequency and quantum parameter on dusty modes. Furthermore, to study the properties of dispersion relation we reduces the above dispersion relation in two limits as follows.

In the hydrodynamic regime, we have assumed that the relaxation time is much less than the $1/\sigma$. The obtained dispersion relation is

\[\left\{ \sigma^{2} + u_{d*,m} \sigma + \frac{\eta k^{2} \sigma}{m_{d} n_{d*}} \right\} \left[ 1 + \frac{J_{0} p_{d*}}{\omega_{n_{d*}}} \right] + \frac{H_{qe}}{k^{2}} \left( 1 + \frac{J_{0} p_{e*}}{\omega_{n_{e*}}} \right) = \omega_{p}^{2} \lambda_{d*}^{2} \left[ (1 - P) + \frac{J_{0} e^{2}}{\lambda_{e} \omega_{e} T_{e0}} \right] \] (9)

which is identical to the result given by Khrapak and Khrapak [9].

Furthermore, in the kinetic limit when $\sigma \tau_{m} \gg 1$, the general dispersion relation (8) yields

\[\left\{ \sigma^{2} + u_{d*,m} \sigma + \frac{\eta k^{2} \sigma}{m_{d} n_{d*} \sigma_{*}} \right\} \left[ 1 + \frac{J_{0} p_{d*}}{\omega_{n_{d*}}} \right] + \frac{H_{qe}}{k^{2}} \left( 1 + \frac{J_{0} p_{e*}}{\omega_{n_{e*}}} \right) = \omega_{p}^{2} \lambda_{q}^{2} \left[ (1 - P) + \frac{J_{0} e^{2}}{\lambda_{e} \omega_{e} T_{e0}} + \frac{J_{0} 4m e^{2}}{\lambda_{e} \omega_{e} h^{2} k^{2}} \right] \] (11)

The above expression shows the combined effects of charge gradient and quantum parameter on the low frequency dusty modes in kinetic regime of strongly coupled dusty plasma along with the polarization force. In absence of strong coupling and charge gradient force Eq. (11) becomes similar to the dispersion relation (22) obtained by Sharma and Jain [11] with plasma approximation.

**NUMERICAL RESULTS**

In the previous section the analytical discussion of the dispersion relation is presented. Now, in order to study the effect of considered parameters on the modes of propagation numerically, Eqs. (9) and (11) are rewritten in the normalized form using the following dimensionless parameters

\[u_{d*} = \frac{V_e e}{\omega_{pd}}, \sigma^{*} = \frac{\sigma}{\omega_{pd}}, \tau_{m}^{*} = \frac{\tau_{m}}{\tau_{m}}, k^{*} = k \lambda_{d*}, H^{*}_{qe} = \frac{H^{*}_{qe}}{\lambda_{D}^{2}}, \omega^{*} = \frac{V_{d}}{\lambda_{D}}, \eta^{*} = \frac{\eta_{i}}{m_{d} n_{d0} \omega_{pd} \lambda_{D}^{2}} \]

The normalized form of Eqs. (9) and (11) are

\[\left\{ \sigma^{2} + u_{d*,m} \sigma^{*} + \frac{\eta k^{2} \sigma^{*}}{k^{2}} \right\} \left[ 1 + \frac{J_{0} y}{\omega_{n_{d*}}} \right] + \frac{H^{*}_{qe}}{k^{2}} \left( 1 + \frac{J_{0} y}{\omega_{n_{c}}} \right) = \left[ (1 - P) + P^{c} + P^{c} \right] \] (12)

and

\[\left\{ \sigma^{2} + u_{d*,m} \sigma^{*} + \frac{\eta k^{2} \sigma^{*}}{k^{2}} \right\} \left[ 1 + \frac{J_{0} y}{\omega_{n_{d*}}} \right] + \frac{H^{*}_{qe}}{k^{2}} \left( 1 + \frac{J_{0} y}{\omega_{n_{c}}} \right) = \left[ (1 - P) + P^{c} + P^{c} \right] \] (13)

where $\xi_{e} = 1 + (J_{0}/\Omega_{s})(n_{d0}/n_{e}), \xi_{m} = 1 + (J_{0}/\Omega_{s})(n_{d0}/n_{e}), P^{c} = (J_{0} e^{2}/\lambda_{e} \Omega_{s} T_{e0})$ and $P^{c} = (J_{0} e^{2}/\lambda_{e} \Omega_{s} T_{e0})$.
In order to discuss the low frequency dusty modes numerically we have plotted the normalized frequency versus normalized wave number of Eqs. (12) and (13) in Figs. (1) and (2) respectively. Figure (1) and (2) show the effect of polarization parameter on the perturbation frequency of strongly coupled dusty modes in hydrodynamic and kinetic regime respectively. It can be seen from figure that the polarization force increases the perturbation frequency of the considered system. The comparison of hydrodynamic and kinetic regime reveals that polarization force is more influencing in kinetic regime.

**FIGURE (1):** The normalized wave frequency $\sigma^*$ versus normalized wave number $k^*$ for different values of dust polarization parameter in hydrodynamic regime.

**FIGURE (2):** The normalized wave frequency $\sigma^*$ versus normalized wave number $k^*$ for different values of dust polarization parameter in kinetic regime.
CONCLUSIONS

The modes of propagation incorporating the strong correlation and polarization force are investigated in degenerate dusty plasma. Based on normal mode analysis, the dispersion relation is derived and its properties are discussed in hydrodynamic and kinetic regimes. It has been observed that the polarization force shows the growing behavior on low frequency dusty modes. The comparison of hydrodynamic and kinetic regimes reveals that polarization force is more influencing in kinetic regime. The results of the investigation can be used to understand the propagation of dusty modes in various astrophysical situations.

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