# Effect of Kappa Distribution Function on EMIC Instability in CUSP REGION

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**Abstract.** In this study, we have presented some previous work but different investigation of kappa distribution function on EMIC waves in plasma by using the method of particle aspect analysis. The dispersion relation, growth rate and growth length of the EMIC waves are derived. The wave is assumed to propagate parallel to the static magnetic field. The whole plasma is considered to consist of resonant and non-resonant particles. It is assumed that resonant particles participate in energy exchange with the waves, whereas non-resonant particle participate in the oscillatory motion of the wave. The results are interpreted for the space plasma parameter appropriate to the cusp region.

## INTRODUCTION

Cornwall and Schulz (1971) discussed the electromagnetic ion cyclotron instability in the presence of cold plasma and gave analytical expression for the growth rates as a function of wave frequency ω. They also concluded that this instability becomes important when the densities of cold and hot plasma are comparable. Ion-cyclotron electromagnetic instability in the presence of warm and cold plasma was studied in details by Cuperman and Gombroff.

An analysis of electromagnetic waves observed by the satellite DEMETER (detection of electromagnetic emission transmitted from Equatorial Regions).These wave were observed in the ULF range (0-20Hz) in the upper ionosphere during a large magnetic storm. There is evidence that may be related to electromagnetic ion cyclotron (EMIC) waves [Curnwall 1965].

Electromagnetic ion cyclotron waves (EMIC) play an important role in the overall dynamics of space plasmas. Electromagnetic ion cyclotron waves are shear Alfven waves, whose frequency ω, is comparable to the ion-cyclotron frequency, ωci. Electromagnetic ion cyclotron waves (EMIC) waves have a clear peak in the power spectrum at frequencies below the proton gyro frequency and they are often generated by precipitating electrons [Temrin M, 1984; Gustafsson G, 1990]. Electromagnetic ion cyclotron waves can be excited by an anisotropic distribution of energetic ring current ions, with energies near a few tens of kev.

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## BASIC TRAJECTORIES

Taking the particle trajectory in the presence of EMIC waves, the dispersion relation, energy, and growth rate is derived for different distribution indices.

The left handed circularly polarized EMIC wave having angular frequency ω is defined as,

$$B_x = B\cos(kz - \omega t)$$
$$B_y = B\sin(kz - \omega t) \tag{1}$$

When the system is commoving with the wave, the electric field vanishes. Thus the wave magnetic field has the form,

$$B = B_x[\cos kz]x + B_y[\sin kz]y \tag{2}$$

Where the following conditions are valid,

$$Z^{wave} = Z^{lab} - (\omega/\kappa)t \tag{3}$$
$$V^{wave} = V^{lab} - (\omega/\kappa)$$

Since $ck/\omega \rangle\rangle 1$ the magnetic field amplitude may be regarded in both systems identical. Thus the equation of ion motion in the wave is given as,

$$\frac{dv_i}{dt} = \frac{q_i}{m_i c}[(V_i \times B_0) + (V_i \times B)] \tag{4}$$

We take the cylindrical variables in velocity space as

$$V_{ix} = V_{\perp i}\cos\phi$$
$$V_{iy} = V_{\perp i}\sin\phi$$
$$V_{iz} = V_{\amalg i} \tag{5}$$

Then the equation of motion is written as,

$$V_{\perp i} = V_{\amalg i0} + \delta V_{\perp i}$$
$$V_{\amalg i} = V_{\amalg i0} + \delta V_{\amalg i} \tag{6}$$

Where $V_{\Pi i}$ ,$V_{\Pi i}$ initial values at t=0, Substituting eq. (5) in (6) we find the following set of equations,

$$\delta V_{\amalg} = \frac{-hV_{\perp 0}\Omega}{[\kappa V_{\amalg 0} - (\omega - \Omega)]} \times [\cos(kz - \omega t - \psi) - \varepsilon\cos(kz - \omega t - \psi - (kV_{\amalg 0} - (\omega - \Omega))t)] \tag{7}$$

Where -Z = $Z_0$ + $V_\Pi$ t and ψ = $\psi_0$ - ωt for the non-resonant and resonant particles.

## DISTRIBUTION FUNCTION

The distribution function of bi-kappa is used

$$F = \frac{1}{\pi^{1/2}}\frac{\Gamma(k+1)}{k^{3/2}\Gamma(k - 1/2)V_{T\perp i}^2 V_{T\amalg i}^2} \times [1 + \frac{v_\perp^2}{k_B v_{\perp i}^2} + \frac{V_{\amalg}^2(\omega - \Omega_i)^2}{k_B T_{\amalg i}^2}]^{-(k+1)}$$

$$F = \frac{1}{\pi^{1/2}}\frac{\Gamma(k+1)}{k^{3/2}\Gamma(k - 1/2)V_{T\perp i}^2 V_{T\amalg i}^2} \times \left[1 + \frac{V_\perp^2}{k_B V_{\perp i}^2} + \frac{V_{\amalg}^2(\omega - \Omega_i)^2}{k_B T_{\amalg i}^2}\right]^{-(k+1)}$$

$$V_{T\perp}^2 = \left[\frac{k - 3/2}{k}\frac{2T_\perp}{m_i}\right]$$

and

$$V_{T\amalg}^2 = \left[\frac{k - 3/2}{k}\frac{2k_B T_{\amalg}}{m_i}\right]$$

The kappa distribution function

$$z_k(\xi) = \frac{1}{\pi^{1/2} k^{1/2}} \frac{\Gamma(k+1)}{\Gamma(k-1/2)} \int_{-\infty}^{\infty} \frac{dx\,(1+x^2/k)^{-k}}{(x-\xi_k)}$$

Where $\xi = \left(\frac{\omega - \Omega_i}{kvT_{\parallel i}}\right)$

## Dispersion Relation

We consider the cold plasma dispersion relation for the EMIC wave as: S. Cuperman and L. Gombroff,

$$\frac{c^2 k^2}{\omega^2} = 1 + \frac{\omega_{pi}^2}{\Omega^2}\left(1 - \frac{\omega}{\Omega}\right)^{-1}$$

Where

$$\omega_{pi}^2 = \frac{4\pi N_0 e^2}{m_i}$$

## Growth Rate

The growth /damping rate $\gamma$ is described as:

$$\frac{\partial U}{\partial t} = 2\gamma U$$

where

$$\frac{d\omega_r}{dt} = -\frac{2\,\partial U}{\partial t}$$

and $\frac{d\omega_i}{dt} = -\frac{\partial U}{\partial t}$, U is energy.

Hence the growth rate is obtained as-

$$\frac{\gamma}{\omega} = \frac{\frac{\Omega}{kvT_{\parallel}}\left[\left(1 - \frac{\omega}{\Omega_i}\right)\frac{T_{\perp i}}{T_{\parallel i}} - 1\right] \times \left[1 + \left(\frac{\omega - \Omega_i}{kvT_{\parallel}}\right)^2\right]^{-(k+1)}}{\left(\frac{ck}{\omega_{pi}}\right)^2 \left(\frac{2\Omega - \omega}{\Omega - \omega}\right) + \frac{1}{2}\frac{\omega^2}{(\Omega - \omega)^2}}$$

## CONCLUSION

In this work, we have conducted a comprehensive mathematical analysis. The effect of electromagnetic ion cyclotron waves with kappa distribution and plasma density in the cusp region is discussed. Our analysis may be useful to explain heating of $He^+$, $O^+$ ions by particle aspect approach along with dynamics of EMIC instability.

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