Relativistic Self-Induced Transmission of Gaussian Laser Beam through High Density Plasma

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Abstract. This paper presents an analytical investigation of self-induced transmission of Gaussian laser beam propagating through high density plasma. Relativistic transmission of a Gaussian laser beam incident normally on a plane interface of a nonlinear and nonabsorbing high density plasma with an intensity dependent dielectric constant has been formulated. The nonlinearity arising from the relativistic variation of mass and the Lorentz force on electrons is considered. Based on Wentzel–Kramers–Brillouin (WKB) and paraxial ray approximation the phenomenon of nonlinear propagation of the transmitted laser radiation has been analyzed for the arbitrary magnitude of nonlinearity. Change in the intensity distribution along the wave front of the Gaussian beam, due to refraction at the interface has also been taken into account. The variation of beamwidth parameter with distance of propagation, self-trapping condition and critical power have been evaluated. Numerical estimates for typical parameters of relativistic laser plasma interaction process indicate the refraction at the interface to have a significant effect on beam transmission.

INTRODUCTION

New short pulse laser technology has recently made possible the production of extremely intense laser sources at the multiterawatt level. Such laser systems are capable of producing focused intensities of about $10^{18}$ W/cm² and above. Electrons oscillating in the ultraintense laser field become relativistic. This makes possible the investigation of entirely new class of physical effects such as relativistic self-focusing of the laser pulses in the plasma, harmonic generation of the laser field by relativistic electrons, inverse Faraday Effect which is expected to produce pulsed magnetic fields of a few hundred Tesla and related phenomena [1-5].

An electromagnetic wave incident on a plasma, or dielectric, slab will be partly reflected and partly transmitted [6,7]. The anomalous transmission of high-power lasers in overdense plasmas has been a problem of considerable interest, due to its relevance for laser-driven fusion and laser ablation of materials. Early studies focused on laser propagation at nonrelativistic intensities. Previously observed experimentally the anomalous transmission of a ring-shaped CO₂ laser through an overdense low-temperature Z-pinch plasma[8]. Other [9] explained these results as plasma cavitation caused by Ohmic heating of electrons and subsequent ambipolar diffusion of the plasma. A theory of laser self-focusing in dense plasma developed by a similar mechanism [10]. The model of anomalous transparency via the parametric excitation of surface waves developed in year 1981 [11].

In the present paper, we study the transmission of a relativistic laser beam with Gaussian distribution of intensity along its wave front through plasma. The axial portion of the laser beam induces a large oscillatory velocity on electrons, raising their relativistic mass and lowering the plasma frequency. Hereby, an analytical investigation of relativistictransmitted laser radiation in plasmas is formulated. In many situations of interest a Gaussian beam is incident on the nonlinear medium, the Gaussian profile gets modified on account of the dependence of the refractive index of the medium on the beam intensity. In such cases, the distortion from the Gaussian intensity distribution is a function of intensity of the incident beam; the relativistic propagation of such beams has not been studied. During analytical treatment effect of collisions has been neglected, as there is almost no effect by collisions near relativistic intensity considered here. In this way, this paper presents an analysis of the relativistic propagation of a Gaussian
laser beam, transmitted across a linear medium/plasma interface for arbitrary magnitude of nonlinearity. The nature of the intensity profile of the transmitted beam has been explored. Based on WKB and paraxial approximation, the propagation equation of such laser beams in a nonlinear medium with arbitrary large magnitude of nonlinearity has been studied. Expressions for the effective dielectric function of plasmas, taking into account relativistic variation of mass is evaluated. An equation for the variation of beam width parameter of the transmitted radiation with distance of propagation has been set up. Numerical computations for a typical set of parameters are performed, followed by results and discussions.

**NATURE OF TRANSMITTED BEAM**

Consider a Gaussian laser beam propagating in the \( z \)-direction through vacuum and normally incident on the plane interface \( (z = 0) \) of the vacuum and the non-absorbing plasma of intensity dependent dielectric constant. The electric field in the vacuum (medium 1) is

\[
E(1) = E_0 e^{i(\omega - \kappa_0 z)} + E_0 e^{-i(\omega - \kappa_0 z)}
\]

and the plasma (medium 2) is

\[
E(2) = E_z(r, f)e^{i[\omega_f - \kappa_2 z]}
\]

where,

\[
k_0 = \frac{\alpha}{c}, \quad k = k_0 \sqrt{\epsilon} \quad \epsilon = \epsilon_0 + \phi(E_z^2)
\]

\( \omega \) and \( \phi \) are linear and nonlinear part of dielectric constant respectively, \( \omega \) is the wave frequency and \( f \) is a function of \( z \). Using the boundary conditions that \( E \) and \( (dE/dz) \) are continuous at \( z = 0 \), with the above equations the usual relations for reflection and transmission are obtained as

\[
R = E_t / E_i = \frac{1 - \sqrt{\epsilon}}{1 + \sqrt{\epsilon}}
\]  

and

\[
T = E_z / E_0 = \frac{2}{1 + \sqrt{\epsilon}}
\]  

The parallelism in the incident beam, corresponding to \( (df/dz) = 0 \), has been assumed. It should be remembered that \( E_0, E_i, \) and \( E_z \) are functions of \( r \), where \( r \) is the radial coordinate of the cylindrical coordinate system. The initial intensity distribution of the incident beam at \( z = 0 \), can be described by

\[
E_0 E_i^* \bigg|_{r=0} = E_0^2 e^{i[r_0/(\epsilon)]}
\]

where, \( r_0 \) is the initial beam width. Expanding \( T \) around \( r = 0 \) by the Taylor expansion one can write

\[
T(r) = a \left[ 1 + (B / a)(r / r_0)^2 \right]
\]

where, \( a = 2 / (1 + \sqrt{\epsilon}(0)) \), \( \epsilon(0) = \epsilon(r = 0, z = 0) \), and \( B \) is a constant which depend on the type of nonlinearity. The values of \( B \) and \( \epsilon(0) \) have been derived in the subsequent sections. The amplitude \( E(2) \) of the transmitted beam at \( z = 0 \) can thus be described as

\[
E(2) E_z^* \bigg|_{r=0} = E_0^2 e^{i[r^2 / r_0] a \left[ 1 + (B / a)(r / r_0)^2 \right]}
\]

(7)

The intensity of the electromagnetic beam inside the nonlinear medium \( |8(\epsilon / 8\pi)\sqrt{\epsilon}|E(2)\|^2 \). Hence Equation (7) also provides a representation of the intensity distribution. It must be pointed out that the intensity profile as given by Equation (7) is correct only up to terms \( (r^2 / r_0^2) \). In fact, in the present work, we have presented an analysis of self-focusing using the paraxial ray approximation and, therefore, we require the intensity profile only near the axis of the beam where \( (r^2 / r_0^2) << 1 \). In this region, since \( (r^2 / r_0^2) << 1 \), contribution of higher order terms to the intensity profile are negligible, hence, the expression of \( E(2) E_z^* \) given by Equation (7) gives a fairly good picture of the self-focusing phenomenon, and a precise intensity expression of the transmitted beam incorporating higher order terms of \( (r^2 / r_0^2) \) is not required.

**PROPAGATION EQUATION OF THE TRANSMITTED BEAM**

The wave equation governing the electric vector of the transmitted beam in the nonlinear medium can be written as

\[
\nabla^2 E(2) - \nabla \left[ \nabla \cdot E(2) \right] + \frac{\alpha^2}{c^2} \epsilon E(2) = 0
\]  

(8)
In the WKB approximation of the second term on the left hand side of the above equation can be neglected, thus

$$V^2E(2) + \frac{\omega^2}{c^2}aE(2) = 0$$

(9)

It may be mentioned that neglecting the term $V(V \cdot E)$ in Equation (8) is justified when $(c^2/\omega^2) |\ln \varepsilon| << 1$. For the set of parameter considered in the present work this inequality is valid. The nonlinear dielectric constant of the medium can, in general, be written as [12]

$$\varepsilon[E(2)E^*(2)] = \varepsilon_0 - \varepsilon_1(f)$$

(10)

here, $f$ is the dimensionless beamwidth parameter defined later in Equation (15). Using the WKB approximation and following [13] one can write

$$E(2) = A(r, z) \left[ \frac{k(0)}{k(f)} \right]^{1/2} \exp \left[ i \frac{\omega}{c} \int k(f) df \right]$$

(11)

where

$$k(f) = \frac{\omega}{c} \left[ \varepsilon_0(f) \right]^{1/2}$$

and

$$k(0) = \frac{\omega}{c} \left[ \varepsilon_0(f = 1) \right]^{1/2}$$

Substituting for $E(2)$ and $\varepsilon$ in Equation (9) one obtains

$$2ik(f) \left( \frac{\partial A}{\partial z} \right) + \left( \frac{\partial^2 A}{\partial r^2} \right) + \frac{1}{r} \left( \frac{\partial A}{\partial r} \right) + \frac{\omega^2}{c^2} \varepsilon_0(f) r^2 A = 0$$

(12)

Putting $A = A_0(r, z) \exp[-iS(r, z)]$ in Equation (12) and separating real and imaginary parts one obtains,

$$2i \left( \frac{\partial A_0}{\partial z} \right) + \left( \frac{\partial^2 A_0}{\partial r^2} \right) + \frac{\omega^2}{c^2} \varepsilon_0(f) r^2 A_0 = \frac{1}{k^2(f)} \frac{\partial^2}{\partial r^2} \left( \frac{\partial A_0}{\partial r} \right)$$

(13)

and

$$\frac{\partial^2 A_0}{\partial r^2} + \frac{\partial S}{\partial r} \frac{\partial A_0}{\partial r} + \frac{1}{r} \frac{\partial S}{\partial r} = 0$$

(14)

The solution of Equations (13) and (14) satisfying the initial condition for the intensity distribution of a Gaussian beam in a plasma at $z = 0$ follows

$$A_0^2 = \frac{E_0^2}{f} \exp \left[ -r^2 / r_0^2 f^2 \right] \left[ 1 + (B / a)(r / r_0 f)^2 \right]^2$$

(15a)

$$S = \frac{r^2}{2} \beta(z)$$

(15b)

and

$$\beta = \frac{\omega}{c} \varepsilon_0^{1/2} \frac{1}{d} \frac{df}{dz}$$

(15c)

here, in the above set of Equations (15), $\beta$ represents the inverse of the radius of curvature of the wave front and $(r_0 f)$ is the width of the beam. In the geometrical optics approximation, $r = r_0 f(z)$ represents a ray in a plane containing the $z$-axis; hence $(df/dz) = 0$ refers to a parallel beam. We substitute for $S$ from Equation (15) in Equation (13) and making use of the paraxial ray approximation i.e., $(r / r_0 f)^2 << 1$. Finally we equate the coefficients of $r^2$ on both sides of the resulting equation and substituting for $\beta$ one obtains

$$\frac{d^2 f}{dz^2} = \frac{1}{k^2(f)r_0^2 f^2} \frac{\omega^2}{c^2} \varepsilon_0(f) r_0^2 f / k^2(f)$$

(16)

Physically, Equation (16) governs the variation of the beam width parameter $f$ with distance of propagation. The first term on the right hand side corresponds to diffraction divergence of the beam and second term corresponds to convergence due to nonlinearity. $\varepsilon_0(f)$ in Equation (16) is the nonlinear dielectric function of the transmitted beam due to relativistic variation of mass, the expression for the same follows. Following [12] the dielectric function due to relativistic variation of mass for a transmitted beam can be written as

$$\varepsilon[E(2)E^*(2)] = \varepsilon_0 + \phi [E(2)E^*(2)]$$

(17)

and
\[
\phi[E(2)E^*(2)] = \frac{\alpha^2}{\alpha^4} \left[ 1 + \left[ 1 - 1/2 \alpha E(2)E^*(2)C_\ell \right]^{1/2} \right]
\]

is the nonlinear dielectric function, where \( \varepsilon = \left[ 1 - \left( \omega^2 / \omega_0^2 \right) \right] \) is the linear part of dielectric function.

Substituting \( E(2) \) from Equation (11) and \( S, A_0 \) and \( \beta \) from Equation (15) in Equation (17) and using paraxial ray approximation, \( \phi \) can be written as

\[
\phi[E(2)E^*(2)] \approx \left[ \frac{k(0) a^2 E_{00}^2}{k(f) 2f^2} \right]^{-3/2} \times \left[ 1 + \frac{k(0) 1}{k(f) 16 \lambda^2 f^2} \right] \times \left[ \frac{1 + k(0) 1}{k(f) 16 \lambda^2 f^2} \right]^{-1/2}\]

Correct in terms in \( r^2 \), where

\[
\phi\left( \frac{k(0) a^2 E_{00}^2}{k(f) 2f^2} \right) = \frac{\alpha^2}{\alpha^4} \left[ 1 - \left[ 1 + \frac{k(0) 1}{k(f) 16 \lambda^2 f^2} \right] \right]^{-1/2}\]

Equation (10) can be put in the convenient form as,

\[
\varepsilon_0(f) = \varepsilon_0 + \frac{\alpha^2}{\alpha^4} \left[ 1 - \left[ 1 + \frac{k(0) 1}{k(f) 16 \lambda^2 f^2} \right] \right]^{-1/2}\]

and

\[
\varepsilon_1(f) = \left[ 1 - 2B/a \alpha \frac{\alpha^2}{\alpha^4} \frac{k(0) a^2 E_{00}^2}{k(f) 4\lambda f^4} \right] \times \left[ 1 + \frac{k(0) 1}{k(f) 16 \lambda^2 f^2} \right] \times \left[ 1 + \frac{k(0) 1}{k(f) 8 \lambda^2 f^2} \right] \times \left[ 1 + \frac{1}{4 \lambda f^2} \right]^{-1/2}\]

here,

\[
B = \left\{ \frac{\alpha^2}{\alpha^4} \frac{a^2 E_{00}^2}{4\lambda f^2} \left[ 1 + \frac{a^2 E_{00}^2}{2f^2} \left[ 1 + \frac{1}{16 \lambda^2 f^2} \right] \right] \right\}^{-1/2}
\]

and

\[
\varepsilon(0) = \varepsilon_0 + \frac{\alpha^2}{\alpha^4} \left[ 1 - 2 \alpha a E_{00} C_\ell \right]^{-1/2}\]

Substituting for \( \varepsilon_0(f) \) and \( \varepsilon_1(f) \) from Equation (20) in Equation (16) the equation governing the beam width parameter is seen to be

\[
\frac{d^2 f}{dz^2} = \frac{1}{k(f) r_0^2 f} - \frac{\alpha^2}{\alpha^4} \frac{a^2 E_{00}^2}{k(f) c^2} \left[ 1 - \frac{2B}{a} \right] \times \left[ 1 + \frac{k(0) 1}{k(f) 16 \lambda^2 f^2} \right] \times \left[ 1 + \frac{k(0) 1}{k(f) 8 \lambda^2 f^2} \right] \times \left[ 1 + \frac{1}{4 \lambda f^2} \right]^{-1/2}\]

For an initial plane wave front of the beam, the initial conditions on \( f \) are \( f(z = 0) = 1 \) and \( df/dz \big|_{z = 0} = 0 \), when the two terms on the right hand side of Equation (21) cancel each other at \( z = 0, \) \( d^2 f/dz^2 \big|_{z = 0} = 0 \), since \( df/dz \) is also zero, and \( f = 1 \) at \( z = 0, \) \( f = 1 \) for all values of \( z \). In other words, the beam propagates without convergence or divergence. The condition for self-trapping is therefore

\[
\left( \frac{\omega_0^2 r_0^2}{c} \right)^2 = 4 \left( \frac{1 + \alpha a E_{00}^2 / 2}{\alpha^2 / 4 \lambda^2 f^2} \right) \left( 1 + \frac{1}{2B/a} \right) \left( a E_{00}^2 / 8 \lambda^2 f^2 \right) \left( 1 + \frac{1}{4 \lambda f^2} \right)^{3/2}
\]

where, \( B_{cr} \) is the value of \( B \) at the threshold. The critical power required of the beam in the nonlinear medium is

\[
P_{cr} = \frac{\varepsilon_0}{8\pi} \int \left| E(2) \right|^2 2\pi dz = \frac{1}{8} \left( \frac{c r_0^2 E_{00}^2}{\alpha^2} \right) \left( 1 + \frac{2B}{a} + \frac{B^2}{a^2} \right)
\]
Critical power of the laser beam $P_{L(cl)}$ in vacuum can be written as,

$$P_{L(cl)} = \frac{P_c}{a^2 \sqrt{\varepsilon(0)} \left( 1 + \frac{2B_c}{a} + \frac{B_c^2}{a^2} \right)}$$  \hspace{1cm} (24)

when, only the linear value of $\varepsilon$ is taken into account for transmission coefficient, the self-trapping condition and critical laser power in the vacuum can be obtained by setting $B_c = 0$ and $a = a_0 = 2 / (1 + \sqrt{\varepsilon})$ in Equation (22) and (24) respectively.

**RESULTS AND DISCUSSION**

It is well known that the electron oscillatory velocity at high laser intensity approaches the velocity of light; hence the relativistic effect becomes important so that an intense short pulse laser of a few femtosecond duration can effectively be transmitted through overdense plasma due to relativistic mass variation. In present paper our objective is to study the propagation of transmitted laser radiation in plasma. Equation (16) is the fundamental second order
The differential equation governing the nonlinear propagation of laser radiation in plasma. Any discussion of the focusing necessitates knowledge of the laser parameter and this is true for the analysis reviewed here. Equation (20) is solved numerically for typical sets of parameters for Nd:YAG laser with irradiance exceeding $10^{18} \text{W/cm}^2$, $a_0 = 0.01 a_0, 0.5a_0, 1.5a_0, 2.0a_0, 2.5a_0$ and $r_0 = (1 - 3) \mu m$.

The variation of critical power $P_c(r)$ of the incident beam with radius is displayed in Figure 1 for overdense plasma. Equation (22) has two roots $E_0 (a_1)$ [corresponding to critical power $P_1 (a_1)$] and $E_0 (a_2)$ [corresponding to critical power $P_2 (a_2)$]; $[P_1 (a_1) < P_2 (a_2)]$. It is seen that $P_1 (a_1)$ increases abruptly with decreasing radius. At radius $< 1.25$, Equation (22) does not have any real root and self-trapping cannot occur. The beam can be self-focused when the power $P$ lies between the two critical powers, namely, $P_1 (a_1) < P < P_2 (a_2)$ and the range $[P_1 (a_1) - P_2 (a_2)]$ increases rapidly with the increasing radius. For case of transmitted radiation ($a = 2/(1 + \sqrt{E_0 (0)})$, $B = B_0$), in overdense plasma ($a_0 = 2.0a_0$) the two critical powers turns out to $P_1 (a_1) = 8.33 \times 10^{18} \text{W/cm}^2$ and $P_2 (a_2) = 120 \times 10^{18} \text{W/cm}^2$ respectively with a very large difference between lower and upper critical power. In this way beam transmission enhanced in overdense plasma due to high beam intensity. Above we have discussed about transmitted radiation. Similarly we can focus for the case of incident radiation at interface ($a = a_0 = 2/(1 + \sqrt{E_0 (0)})$, $B = 0$). Figure 2 and Figure 3 describe the reflection and transmission of incident laser incident in plasma at different plasma density. Coefficient of reflection ranges from 0 to 0.9 and coefficient of transmission ranges from 1 to 2 that shows that transmission is higher as compared to reflection. Figure 4 display variation of beamwidth parameter $f$ with distance of propagation in focusing region for overdense plasma. When the power of the beam $P$ lies between two critical values, the beamwidth parameter oscillates between $f = l$ and another well-defined value. Thus the beam alternately converges and diverges as it propagates in the medium; in other words the medium acts as an oscillatory wave-guide. It is found that qualitative behavior of $f$ with distance of propagation for transmitted radiation ($a = 2/(1 + \sqrt{E_0 (0)})$, $B = B_0$) are similar to the case of incident radiation at interface ($a = a_0 = 2/(1 + \sqrt{E_0 (0)})$, $B = 0$); however the quantitative values are quite different. For transmitted radiation oscillatory focusing and defocusing occurs rapidly in comparison to incident radiation. In defocusing region for high density plasma a convergence is seen, that represent the penetration of transmitted laser radiation in high density plasma.

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